EFFECT OF DESIGN PARAMETERS ON THE STABILITY OF MULTI-MASS GEARED SYSTEMS SUPPORTED ON FLUID FILM BEARINGS AND SUBJECTED TO UNBALANCE AND TORSIONAL EXCITATION

By

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To my parents

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NOMENCLATURE

M_i = mass of rotor i ($i = 1,2,3,4$)
W_i = weight of rotor i ($i = 1,2,3,4$)
J_i = mass moment of inertia of rotor i ($i = 1,2,3,4$)
x_i = displacement of rotor i along x direction($i = 1,2,3,4$)
y_i = displacement of rotor i along y direction($i = 1,2,3,4$)
θ_i = angular displacement of unit i w.r.t. z axis ($i = 1,2,3,4$)
d_t = displacement of tooth spring
d_g = gear whirl
ϕ_i = phase angle of displacement of unit i ($i = 1,2,3,4$)
e_i = position of unbalanced mass of unit i from the center of mass ($i = 1,2,3,4$)
ω_r = rotating frequency of shaft
ω_x = natural frequency of whirl along x direction (simple supports with load in the middle)
ω_y = natural frequency of whirl along y direction
ω_ω = natural frequency of whirl
ω_{no} = nominal natural frequency of gear mesh mode
ω_n = uncoupled natural frequency
ω_{ex} = excitation frequency of torque
τ_r = period of rotation of shaft 1 ($2\pi/\omega_r$)
τ_{no} = period of gear mesh mode ($2\pi/\omega_{no}$)
τ_{ex} = period of torque excitation ($2\pi/\omega_{ex}$)
τ_y = period of whirl along y direction ($2\pi/\omega_y$)
τ_ω = period of whirl ($2\pi/\omega_w$)
k_{pqj} = stiffness coefficient of journal bearings of shaft j along p direction due to displacement along q direction ($p = x,y$; $q = x,y$; $j = 1,2$)
c_{pqj} = damping coefficient of journal bearings of shaft j along p direction due to displacement along q direction ($p = x,y$; $q = x,y$; $j = 1,2$)
k_{tj} = torsional stiffness of shaft j ($j = 1,2$)
f_j = friction coefficient of shaft j ($j = 1,2$)
I_j = area moment of inertia of shaft j w.r.t. x axis ($j = 1,2$)
I_{zj} = area moment of inertia of shaft j w.r.t. z axis ($j = 1,2$)
A_{ij} = influence coefficient ($i = 1,2,3,4$; $j = 1,2,3,4$)
B_{ij} = coefficient related to bearing performance ($i = 1,2,3,4$; $j = 1,2,3,4$)
k_m = mesh stiffness
k_o = nominal mesh stiffness
c_m = mesh damping

ω_m = mesh frequency
ϕ = pressure angle
r_i = radius of rotor i (i = 1,2,3,4)
D_j = diameter of shaft j (j = 1,2)
T_i = torque on rotor i (i = 1,2,3,4)
GR = gear ratio
W_t = width of tooth
δ_i = distance of phase plane instantaneous center from origin
ρ_i = instantaneous radius of arc on phase plane
l_i = instantaneous length of chord on phase plane
φ_i = instantaneous angle of arc on phase plane
E = Young's modulus
G = shear modulus
L_1 = length of shaft 1 between overhang mass and bearing close to it
L_2, L_3 = length of shaft 1 between gear and bearing
L_4 = length of shaft 2 between overhang mass and bearing close to it
L_5, L_6 = length of shaft 2 between gear and bearing
L = total length of shaft 1 (=L_1 + L_2 +L_3)
c = clearance of oil film bearing
μ = relative oil viscosity
$\mu(T)$ = oil viscosity at temperature T
z = number of teeth
accr. cr. = predetermined accuracy criterion for phase plane calculation

Abstract of Dissertation Presented to the Graduate
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EFFECT OF DESIGN PARAMETERS ON THE STABILITY OF MULTI-MASS GEARED SYSTEMS SUPPORTED ON FLUID FILM BEARINGS AND SUBJECTED TO UNBALANCE AND TORSIONAL EXCITATION

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Many high speed mechanical systems incorporate gearing for speed reduction. This study investigates the stability of multi-rotor geared systems supported on oil film bearings taking into consideration the coupling between torsional and lateral dynamics.

The emphasis of the study is on the analysis of the interaction between the combined torsional and whirl instabilities.

The result of simulation in this study checked the previously published empirical approach for predicting nonharmonic whirl of geared systems which results from the inertial coupling at the gear mesh.

The feasibility of inducing a lateral and the torsional instability to neutralize an anticipated unstable condition is investigated. The possibility of suppressing the instabilities by controlling the parameters of the oil film bearings is also considered.

It is interesting to note that the coupling between the torsional and lateral oscillations can induce torsional instability due to unbalance excitation. Similarly, a torsional disturbance can induce whirl.

A procedure is given to guide the designer in selecting the system parameters which insure its safe operation in critical situations where instabilities or resonances are expected.

It is shown that torsional instability can be avoided by inducing a tuned whirl using an unbalance excitation in a system supported on either ball bearings or oil film bearings.

CHAPTER 1
INTRODUCTION

Gears and Geared Systems

Gears, which have been in use for over three thousand years, are one of the oldest and most widely used machine elements. They proved over the years to be the most important means of transmission because of their ability to transform wide ranges of power and speed at a high efficiency.

Historically, gear design has been more art than science, but this has changed rapidly in recent years to become an important scientific discipline. In their earlier use, the main requirement in the design of a geared system was the ability to carry the necessary load at the desired speed ratio. The load carrying capacity determines the size of the tooth, while the speed ratio is satisfied by the relative number of teeth in the matching pairs. Because of the early slow speed application of gears, no attention was given to the instantaneous uniformity of the speed.

The increase in the speed of operation, however, requires consideration of numerous problems such as profile geometry, tooth contact condition, fluctuation of mesh stiffness, surface roughness, and oil film thickness.

In many cases, gear design is the province of a specialist. Nonetheless, the design or specification of a gear train is only part of overall gear system design.

Gear System Dynamics

Gear system dynamics is a problem which has received considerable attention due to its practical importance.

There are numerous sources of excitation in geared systems which include errors in manufacturing and assembly as well as the time-varying nature of mesh stiffness which is inherent to the transmission of motion between the mating teeth. These disturbances manifest themselves in torsional as well as lateral oscillation which can produce resonances and instabilities even under conditions of constant applied loads and speeds.

Instability can be viewed as the result of negative damping creating a build-up in the system oscillation. The rate of build-up is generally exponential in nature, and consequently, very high amplitudes of oscillation can be reached in relatively short terms. In the case of geared systems, instabilities can occur in the form of rotor whirl (lateral oscillation) at certain rotational speeds as well as torsional oscillations at particular mesh and excitation frequencies.

Torsional Dynamics of Gear Systems

Torsional vibration of gear systems has been extensively treated in the literature [1-21]. It is commonly accepted that there are several sources of torsional vibration in geared systems. These excitations can be classified as internal disturbances which are inherent to the system due to the nature of the gear mesh, and external disturbances resulting from fluctuations in the input and output loads or speeds. Excitations resulting from tooth engagement include the effect of tooth error, momentary separation of the teeth, and variation of mesh stiffness.

Houser and Seireg [1] evaluated experimentally the effect of gear system parameters on tooth loads at speeds away from system resonances. In order to observe the dynamic phenomena in gear teeth beyond statistical variations, large pitch errors were introduced to insure that the dynamic data were sufficiently magnified. Large variations in face width were also introduced to simulate significant changes in contact patterns. The study shows the result of the test dealing with face width variation in both a spur gear and a helical gear revealed no noticeable speed effect on the strains developed in the gear teeth. The tests involving large manufactured pitch errors showed significant dynamic strain effects which appear to be approximately linear with speed and independent of load for both spur and helical gears.

An important source of dynamic excitation in gear systems is the variation in mesh stiffness due to the changes in length of contact. This change, in the case of spur gears, is inherent in the gear operation due to the transmission from double to single tooth contact. Other sources which contribute the variation in mesh stiffness are changes in load distribution and change in contact patterns. These may result from bearing clearances, shaft and housing deflections, or manufacturing errors. The variation of tooth stiffness of a pair of matching gears gives rise to a periodic excitation at the gear mesh which results in dynamic response of the gear system. The oscillatory tooth forces may attain large magnitude to cause failure of teeth. Torsional resonance in gear systems can occur at frequencies, other than the nominal natural frequency, which are called subharmonic and superharmonic resonances [2, 3].

There have been extensive studies for the prediction of regions of system resonance and instability for geared systems with variable mesh stiffness [2-5].

The accurate evaluation of the actual pattern of stiffness variation in a gear system is a difficult problem. The compliance of the mesh is influenced by the position of the point of contact on the tooth profile, variation in the length of contact, shaft misalignment, load distribution, and deformation of the wheel body.

Bollinger and Harker [4] considered the effect of harmonically varying mesh stiffness on the system stability and calculated the necessary amount of damping for stable operation.

Another important aspect of the torsional dynamics of geared systems is the resonant behavior of the pinion-gear system under steady state operating conditions.

Benton and Seireg [2] presented a computer simulation procedure for predicting the steady state response, resonances, and instabilities of pinion-gear system subjected to sinusoidal torsional excitation. Their study isolated the gear pair from the rest of the system and treated it as uncoupled from the other connected elements of the total drive based on the assumption that mesh of gear pair has high stiffness and low effective inertias. Sinusoidal and square wave variations of the mesh stiffness were considered to investigate the dynamics of the system under different speeds when subjected to uniform and cyclic loading conditions. A response spectrum of a simple model of a pinion-gear pair subjected to harmonic excitation was presented. The results which were correlated with experimental data show the influence of the harmonic excitation on the overall response of the system. The different models of stiffness variation used showed considerable influence on the dynamic behavior of pinion-gear systems. Variations of mesh stiffness affects both the frequencies and the pattern of the geared system resonances and instabilities.

Benton and Seireg [3] extended their research on torsional vibration of pinion-gear system to investigate the parameters causing high dynamic magnifications and instabilities in multiple reduction geared systems. These factors include the effect of external excitations, system inertia, variation in mesh stiffness, contact ratio, and damping in the mesh. The study presented a procedure which can be applied to multiple reduction gear drives to aid in the analysis and design of such systems when the exact nature of stiffness variation is not precisely known. The influence of the system inertia, variation in mesh stiffness, contact ratio, and dissipation in each mesh on the gear system performance is also discussed.

Tooth backlash can also be a contributing factor in the vibrations of geared systems. Azar and Crossley [5] examined these effects both experimentally and analytically in an attempt to predict the vibrations observed in the driven shaft of a lightly loaded gear system. They reported the presence of large vibrations which are influenced by the tooth backlash in the driven shaft when the mesh frequency is one half the torsional natural frequency of the driven shaft system. A digital simulation model, in the study, is developed to represent a lightly loaded geared torsional system consisting of a drive unit, gear pair, and load connected by flexible shafts.

Problem of tooth separation and impact in a geared system has also been subjected to considerable attention [5-10]. Innoue and Seireg [6] presented a digital simulation of the vibration and the impact in a dual drive branched gear system during the engagement and disengagement of the prime mover. The study illustrated the existence of an optimal backlash condition for the idle pinion which minimizes the dynamic tooth load. Seireg, Shah, and Khazekhan [7] investigated the effect of gear vibration through the backlash on the dynamic tooth load experimentally.

Other factors which place a penalty on the load carrying capacity of gears are the manufacturing errors. The types of errors and their effects on the system dynamics can be found in previous works [11-13].

There are many practical situations where resonances and instabilities in pinion-gear systems are difficult to predict in the design stage due to the unreliability of estimating the mesh stiffness and damping parameters.

In an attempt to insure the stability and safe performance of pinion-gear systems, Benton and Seireg [14] investigated the feasibility of the design of vibration absorber for gear drives which can provide an appropriate solution for such conditions. They developed the dynamic equations of motion for the absorber system to determine the pertinent variables necessary for general description of the absorber system. The conditions for uncoupling these equations are determined, and the design parameters to be considered in the optimization are specified. A search is undertaken to evaluate the optimal parameters for different gear system conditions and the results are given as design charts. The study shows that an optimal dynamic absorber can be designed which is capable of providing safe operation of pinion-gear system at all speeds including unexpected resonance and instability conditions.

Lateral Dynamics of Gear Systems

A considerable number of studies of gear system whirl have also been reported. A review of the problem and developments in the theory of whirl can be found in previous works [22-44]. Some of the recent contributions to the analysis of whirl are the effects of gear constraint on the whirl of simple supported pinion shafts [22], the effects of the change in the cross section [23-25], and the effects of shaft support [26, 27].

It was reported that the effect of the gear restraint on the whirl of the pinion shaft is a reduction of the whirl amplitude, a change in the resonant frequency, and the appearance of subharmonic as well as superharmonic resonances [22].

A simulation of the whirl interaction at the gear mesh is investigated, in which the pinion-gear system is modeled as a coupled two rotor problem [28]. The gear rotor, in the study, is assumed to be supported on isoviscous fluid bearings and the transmitted gear load is taken into consideration. Each rotor is simulated as a two degree of freedom system coupled at the point of contact during the mesh. The result of the study shows that the gear constraint with zero transmitted load causes an increase in the threshold speed. Instability in this case occurs, as expected, in both the pinion and the gear rotors. The effect of the transmitted load is also shown to increase the threshold speed with one of the two rotors reaching instability first and therefore causing the system instability.

A rotor whirl simulation procedure [29] is described for investigating the effect of the unbalance magnitude on the whirl of rotors. In the study, a rigid rotor on a simplified isoviscous film is considered for illustration in an attempt to present a general simulation procedure which can be used for the analysis of this class of problems.

The designer is generally interested in predicting the expected peak amplitude of the whirl orbit at a given operating speed resulting from a particular amount of unbalance as well as the stability of the rotor operation in the design stage. Some of the contributions to the analysis of this problem are made by Lund et al [30-38].

The effect of fluid film bearing on the whirl has also been widely studied. Seireg and Dandage [39] described an empirical design procedure for predicting the performance of fluid film bearing based on experimental thermohydrodynamic consideration. The study utilized developed relationships to construct design nomograms for evaluating a modified Sommerfeld number which can replace the isoviscous Sommerfeld number in evaluating the performance characteristics of the bearing for any given oil and inlet temperature. The modified Sommerfeld number can then be utilized in the standard formulas to calculate eccentricity ratios, oil flow, frictional loss, and temperature rise, as well as stiffness and damping coefficients for bearings.

It is generally accepted that the dynamic properties of the fluid film can be expressed in terms of four stiffness and four damping coefficients [39, 45, 46]. The investigation of rotor stability involves the computation of the stiffness and the damping coefficients.

Coupling of the Lateral and Torsional Dynamics

It has been common practice for the design procedure of a geared system to analyze the torsional dynamics and the lateral dynamics separately. In this way, the effect of gear coupling is only partially accounted for, and the coupling between torsional and lateral vibrations is neglected. As a result, the procedure fails to account for several important aspects such as the shift in critical speeds and change in mode shapes caused by the coupling, change in the instability threshold, the excitation of all rotors in the system caused by the unbalance in any of the rotors, and the damping in the torsional modes contributed by the lateral motion of the journals in their bearings.

Lund [47] used the impedance matching method [16] in an effort to calculate the coupled torsional-lateral vibrations in a geared system assuming a constant mesh stiffness. The study considered the damped forced vibrations of the system caused by mesh errors and unbalance, whose complex eigenvalues define the damped critical speeds and the instability threshold of the system. The vibratory modes for the rotors supported on fluid film bearings are calculated independently, using the Holzer method for torsional vibrations, and the Myklestad-Prohl method for lateral vibrations. They are subsequently coupled through impedance matching at the gear meshes. The resulting equations are solved for the unknown mesh contact forces, and the roots of the coefficient matrix give the eigenvalues of the system.

Scope of This Work

It can be seen from the literature review that most of the previous studies dealt with either the lateral or torsional dynamics separately for isolated pinion-gear systems. Even in the studies treating the entire geared systems, the investigations of dynamics of the system were confined to specific phenomena. Effects of coupling of torsional-lateral dynamics were treated by Lund [43]. He considered the coupling with the assumption of constant stiffness for the mesh in order to simplify the determination of the coupled frequencies of the system and the threshold of instabilities.

Since the variation of stiffness during the mesh cycle is an inherent characteristic of gear drives, this study is therefore initiated to investigate the combined effects of coupling the torsional vibration with the consideration of time-varying stiffness and lateral dynamics of a multi-mass geared system. The drive shafts are supported on fluid film bearings and the rotors are subjected to unbalance and torque excitations which can be expected in many practical applications. An important objective is to develop rules which can be followed in the design stage in order to insure the safe operation at all operating conditions including resonances and instabilities. Emphasis is placed on the interaction at resonant and unstable regions. The goal is to develop a procedure for selecting the system parameters which insure the stability of the system by taking advantage of the coupling effects.

Since both the lateral and torsional oscillations of a geared system interact at the points of the tooth contact, it is conceptually possible that a system can be designed where the oscillations at the contact point can be made to counteract and vanish by the appropriate selection of the design parameters and the phase relation between the excitations.

A simple supported geared system with two overhang rotors is considered in this investigation. Each rotor has three degrees of freedom. As a result, the system has twelve degrees of freedom. In unstable situations, the response of the system usually exhibits one dominant frequency of oscillation at which build-up occurs.

The possibility of suppressing an instability by inducing another instability of a same type is investigated. The basic concept is extended to see if it is also possible to suppress an instability by inducing another "tuned" instability of a different type. Since the system under consideration has time-varying coefficients and is supported on bearings with non-linear characteristics, exact analytical stability criteria are not practical. A procedure based on numerical solution of the equations of motion is developed with the objective of guiding the selection of the system parameters which insure its stable performance under the expected operating conditions. Because of the numerical nature of the solution, the developed approach can be readily extended to the design of more complex gear systems.

CHAPTER 2
SYSTEM SIMULATION

Physical Model

The systems dealt with in this work are typical geared systems. They represent a single reduction gear unit with and without overhang rotors.

The systems, which are illustrated in Fig. 2.1 and Fig. 2.2, respectively, are composed of a gear pair with the high and low speed shafts simply supported on fluid film bearings. The shafts are assumed to be massless. Each of the gear wheels and the overhang masses is assumed to have three degrees of freedom representing the x and y movements of the center of mass and the angular oscillation, θ about the center axis. Since the analysis is primarily directed toward high speed systems, the outer diameters of the wheels are relatively small and consequently the gyroscopic movement of the gears can be neglected. The system parameters are listed in the Nomenclature.

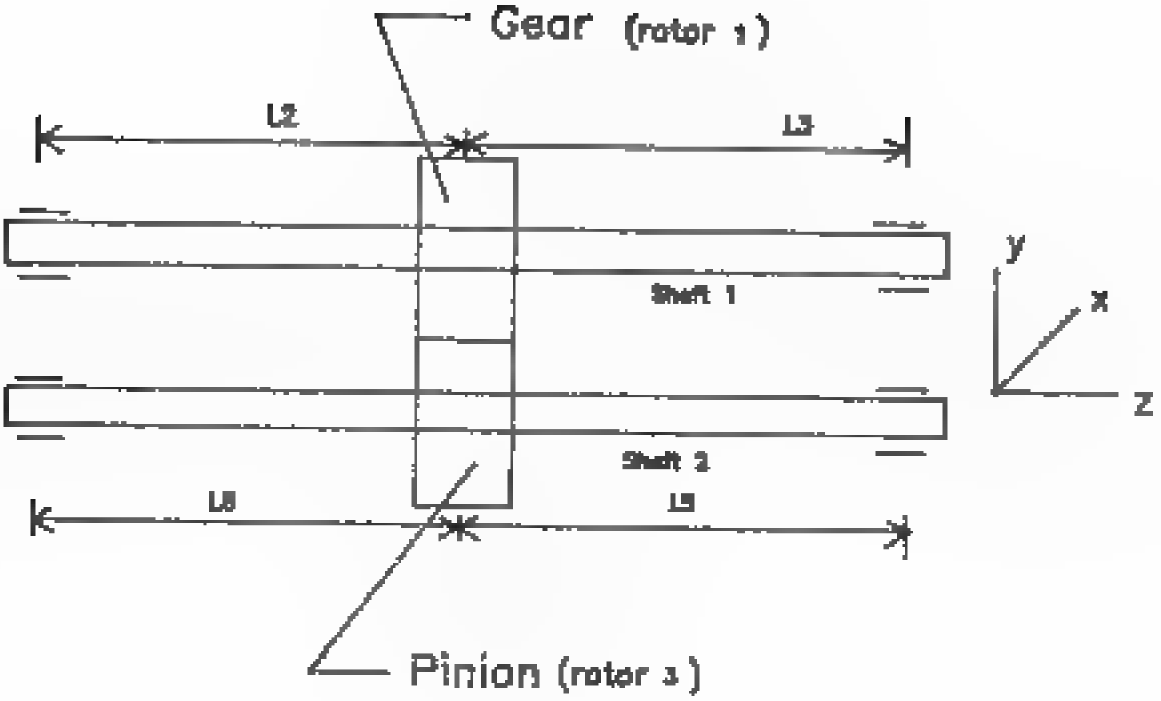


Fig. 2.1 Two Rotor Geared System

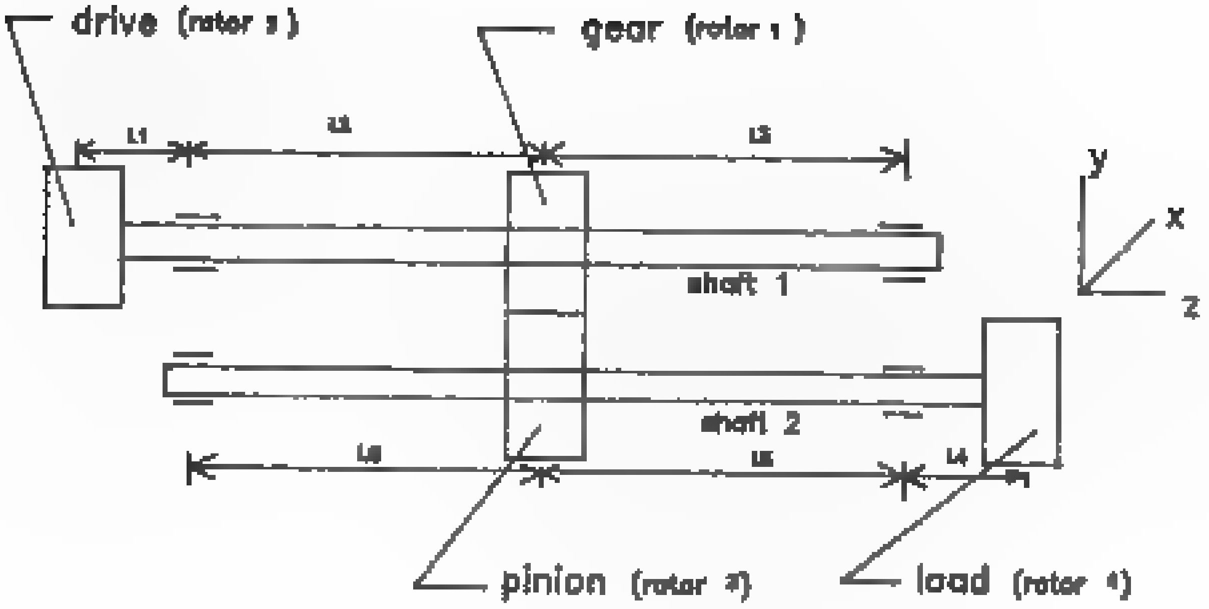


Fig. 2.2 Four Rotor Geared System

System Equations

The equations of motion can be derived in the same way for both systems. Therefore, the equations of motion for the four rotor geared system are obtained, and the two rotor system is considered to be a special case of the general formulation where the overhang masses are zero. Since the general system has twelve degrees of freedom, a 12 x 12 matrix (Fig. 2.3) consisting of twelve equations of twelve variables can be constructed as follows.

The coefficients in the matrix are obtained by assuming a small movement in each degree of freedom and evaluating the forces induced by this movement on the entire system. For example, for element 1, a small positive displacement x_1 is imposed on the system, and the resultant force on rotor 1 along the x direction is obtained to fill that element. The resultant force is evaluated from the reactions at the mesh, the shafts, and the bearings as a response to the movement. For the rest of the elements in the first column, a similar procedure is applied to obtain the forces on all four rotors. Also, the remaining elements in the matrix are determined by evaluating the forces due to the displacements x_2, x_3, ..., θ_4.

	x_1	y_1	θ_1	.	.	.	x_4	y_4	θ_4
$M_1 \frac{d^2x_1}{dt^2}=$	1	13	.	.	.		.	.	133
$M_1 \frac{d^2y_1}{dt^2}=$	2	14	.	.	.		.	.	134
$J_1 \frac{d^2\theta_1}{dt^2}=$	3	15	.	.	.		.	.	135
.	.	.	.	.	.		.	.	.
.	.	.	.	.	.		.	.	.
$J_4 \frac{d^2\theta_4}{dt^2}=$	12	24	.	.	.		.	.	144

Fig. 2.3 A 12x12 Matrix of Coefficients for the Equations of Motion

Force at the Mesh

Fig. 2.4 illustrates the geometry of a pair of gears [48] in mesh. The force induced on rotor 1 due to the displacement x_1 can be expressed as

$$\cos\phi \; [k_m \; (-\cos\phi) \; x_1 + c_m \; (-\cos\phi) \; \frac{dx_1}{dt} \;] \qquad 2.1$$

where

k_m , the time varying stiffness of the mesh

$$= k_o \; [1 - A \cos \; (\omega_m \; t)] \qquad 2.2$$

(k_o: the nominal mesh stiffness)

c_m , the damping coefficient at the mesh

$$= 2 \; \xi \; [k_m \frac{M_1 \; M_3}{M_1 + M_3} \;]^{.5} \qquad 2.3$$

ω_m , the mesh frequency

= (rotating speed of shaft) (number of teeth of gear) 2.4

A , constant representing the degree of fluctuation of stiffness

= .5 for spur gears 2.5

ξ , damping ratio

(assumed to be 0) 2.6

ϕ , pressure angle

(assumed to be 20 degree) 2.7

Force at the Bearings

A diagrammatic illustration of the effect of the oil film bearing support, and the motion in the y-z plane are shown in Fig. 2.5.

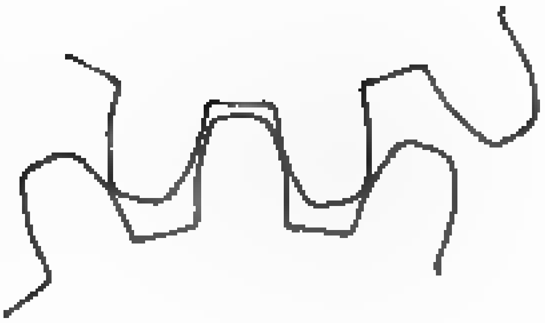
The stiffness and the damping coefficients due to the bearing support is calculated using the subroutine Brng (Appendix) which is based on the work by Seireg and Dandage [39]. The force on rotor 1 can be written as

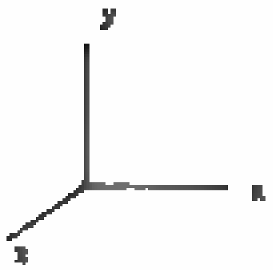
$$- k_{xx1} \frac{L_1^2 + L^2}{(L_1 + L_2)^2} x_1 - c_{xx1} \frac{L_1^2 + L^2}{(L_1 + L_2)^2} \frac{dx_1}{dt} \qquad 2.8$$

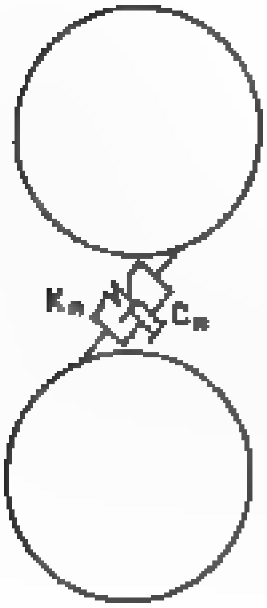
Force due to Bending

As illustrated in Fig. 2.6, the force due to the bending of the shaft [49] as a result of displacement x_1 can be expressed as

$$- \frac{6 E I_1 (L_2 + L_3)}{L_2 L_3 [L_2^2 + L_3^2 - (L_2 + L_3)^2]} x_1 \qquad 2.9$$



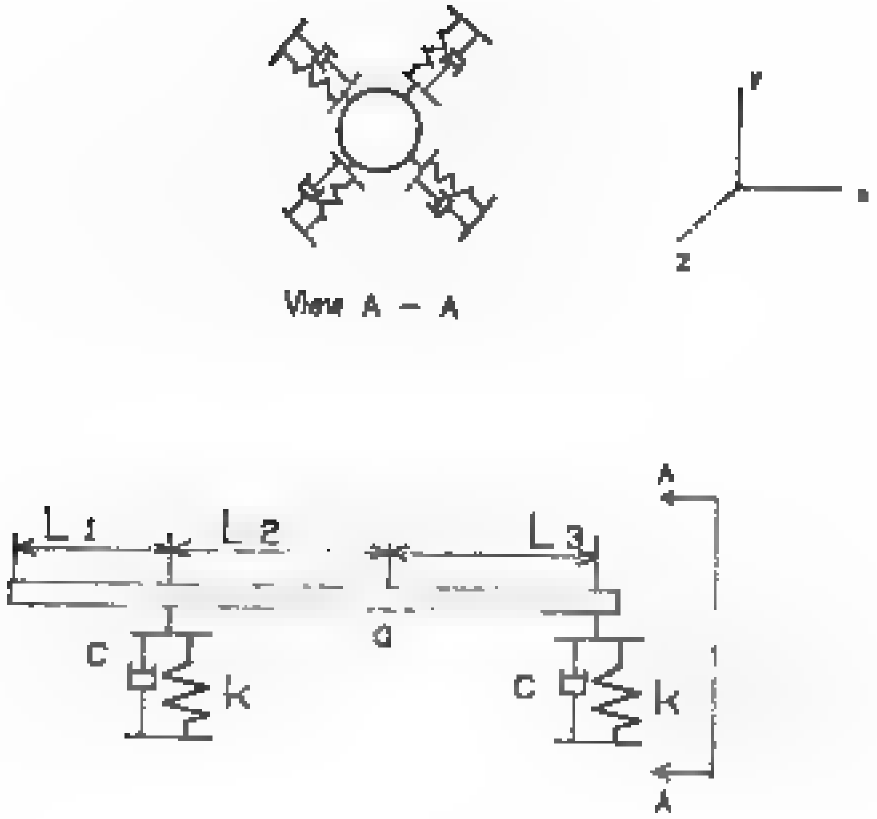




$$k_m = k_o\,[1 - A\cos(\omega_m t)]$$

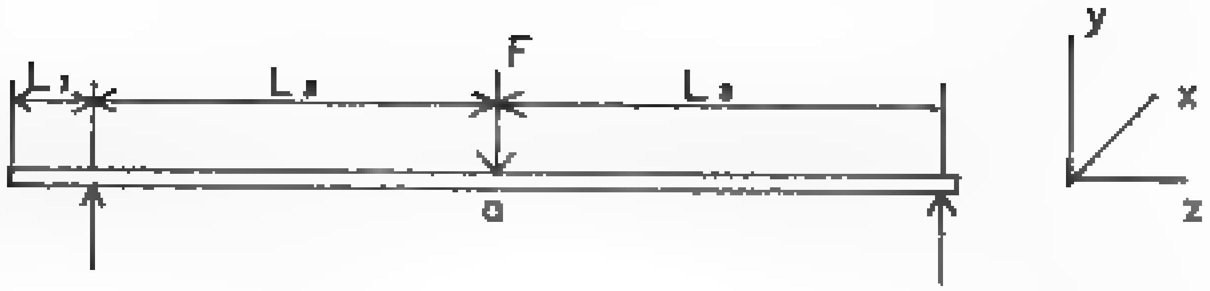
$$C_m = 2\,(\xi)\,\left[k_m \frac{M_1\ M_3}{M_1 + M_3}\right]^{.5}$$

Fig. 2.4 Stiffness and Damping from Tooth Contact



$$k_{eq} = -k\,\frac{L_1^2 + (L_1 + L_2 + L_3)^2}{(L_1 + L_2)^2}$$

Fig. 2.5 Shaft Supported on Oil Film Bearings



$$d_a = \frac{F L_2 L_3 [L_2^2 + L_3^2 - (L_2 + L_3)^2]}{6 E I (L_2 + L_3)} \quad (= F A_{11})$$

Fig. 2.6 Shaft Subjected to Bending

Equations of Motion

The 144 elements of the matrix (Fig. 2.3) can then be used to formulate the following equations of motion for the general system.

$$
\begin{aligned}
M_1 \frac{d^2x_1}{dt^2} = &- x_1 \left(\cos^2\phi \; k_m + k_{xx1} \; B_{11} + \frac{1}{A_{11}}\right) \\
&- \frac{dx_1}{dt} \left(\cos^2\phi \; c_m + c_{xx1} \; B_{11}\right) \\
&- y_1 \left(\cos\phi \; \sin\phi \; km + k_{xy1} \; B_{11}\right) \\
&- \frac{dy_1}{dt} \left(\cos\phi \; \sin\phi \; c_m + c_{xy1} \; B_{11}\right) \\
&- \theta_1 \; r_1 \; \cos\phi \; k_m - \frac{d\theta_1}{dt} \; r_1 \; \cos\phi \; c_m \\
&+ x_2 \left(k_{xx1} \; B_{12} + \frac{1}{A_{12}}\right) + \frac{dx_2}{dt} \; c_{xx1} \; B_{12} \\
&+ y_2 \; k_{xy1} \; B_{12} + \frac{dy_2}{dt} \; c_{xy1} \; B_{12} \\
&+ x_3 \; \cos^2\phi \; km + \frac{dx_3}{dt} \; \cos^2\phi \; c_m \\
&+ y3 \; \cos\phi \; \sin\phi \; k_m + \frac{dy_3}{dt} \; \cos\phi \; \sin\phi \; c_m \\
&- \theta_3 \; r_3 \; \cos\phi \; k_m - \frac{d\theta_3}{dt} \; r_3 \; \cos\phi \; c_m \\
&+ M_1 \; e_1 \; \omega_1{}^2 \; \cos(\omega_1 \; t + \Phi_1) \\
&+ \frac{A_{22}}{A_{12}} \; M_2 \; e_2 \; \omega_1{}^2 \; \cos(\omega_1 \; t + \Phi_2) \qquad 2.10
\end{aligned}
$$

$$\begin{aligned}
M_1 \frac{d^2 y_1}{dt^2} = & - x_1 (\cos\phi \sin\phi\ k_m + k_{yx1} B_{11}) \\
& - \frac{dx_1}{dt} (\cos\phi \sin\phi\ c_m + c_{yx1} B_{11}) \\
& - y_1 (\sin^2\phi\ k_m + k_{yy1} B_{11} + \frac{1}{A_{11}}) \\
& - \frac{dy_1}{dt} (\sin^2\phi\ c_m + c_{yy1} B_{11}) \\
& - \theta_1 r_1 \sin\phi\ k_m - \frac{d\theta_1}{dt} r_1 \sin\phi\ c_m \\
& + x_2 k_{yx1} B_{12} + \frac{dx_2}{dt} c_{yx1} B_{12} \\
& + y_2 (k_{yy1} B_{12} + \frac{1}{A_{12}}) + \frac{dy_2}{dt} c_{yy1} B_{12} \\
& + x_3 \sin\phi \cos\phi\ k_m + \frac{dx_3}{dt} \sin\phi \cos\phi\ c_m \\
& + y_3 \sin^2\phi\ k_m + \frac{dy_3}{dt} \sin^2\phi\ c_m \\
& - \theta_3 r_3 \sin\phi\ k_m - \frac{d\theta_3}{dt} r_3 \sin\phi\ c_m \\
& + M_1 e_1 \omega_1{}^2 \sin(\omega_1 t + \Phi_1) \\
& + \frac{A_{22}}{A_{12}} M_2 e_2 \omega_1{}^2 \sin(\omega_1 t + \Phi_2) \qquad 2.11
\end{aligned}$$

$$\begin{aligned} J_1 \frac{d^2\theta_1}{dt^2} = & - x_1 r_1 \cos\phi\, k_m - \frac{dx_1}{dt} r_1 \cos\phi\, c_m \\ & - y_1 r_1 \sin\phi\, k_m - \frac{dy_1}{dt} r_1 \sin\phi\, c_m \\ & - \theta_1 (k_{t1} + r_1{}^2 k_m) - \frac{d\theta_1}{dt} r_1{}^2 c_m \\ & + \theta_2 k_{t1} + x_3 r_1 \cos\phi\, k_m + \frac{dx_3}{dt} r_1 \cos\phi\, c_m \\ & + y_3 r_1 \sin\phi\, k_m + \frac{dy_3}{dt} r_1 \sin\phi\, c_m \\ & - \theta_3 r_1 r_3 k_m - \frac{d\theta_3}{dt} r_1 r_3 c_m \\ & - T - \mathrm{sgn}(\frac{d\theta_1}{dt}) f_1 W_1 D_1 \end{aligned} \qquad 2.12$$

$$\begin{aligned} M_2 \frac{d^2x_2}{dt^2} = & \quad x_1 (k_{xx1} B_{12} + \frac{1}{A_{12}}) + \frac{dx_1}{dt} c_{xx1} B_{12} \\ & + y_1 k_{xy1} B_{12} + \frac{dy_1}{dt} c_{xy1} B_{12} \\ & - x_2 (k_{xx1} B_{22} + \frac{1}{A_{22}}) - \frac{dx_2}{dt} c_{xx1} B_{22} \\ & - y_2 k_{xy1} B_{22} - \frac{dy_2}{dt} c_{xy1} B_{22} \\ & + M_2 e_2 \omega_1{}^2 \cos(\omega_1 t + \Phi_2) \\ & + \frac{A_{11}}{A_{12}} M_1 e_1 \omega_1{}^2 \cos(\omega_1 t + \Phi_1) \end{aligned} \qquad 2.13$$

$$M_2 \frac{d^2 y_2}{dt^2} = x_1 k_{yx1} B_{12} + \frac{dx_1}{dt} c_{yx1} B_{12}$$

$$+ y_1 (k_{yy1} B_{12} + \frac{1}{A_{12}}) + \frac{dy_1}{dt} c_{yy1} B_{12}$$

$$- x_2 k_{yx1} B_{22} - \frac{\text{[illegible]}_2}{dt} c_{yx1} B_{12}$$

$$- y_2 (k_{yy1} B_{22} + \frac{1}{A_{22}}) - \frac{dy_2}{dt} c_{yy1} B_{22}$$

$$+ M_2 e_2 \omega_1^2 \sin(\omega_1 t + \phi_2)$$

$$+ \frac{A_{11}}{A_{12}} M_1 e_1 \omega_1^2 \sin(\omega_1 t + \phi_1) \qquad 2.14$$

$$J_2 \frac{d^2\theta_2}{dt^2} = \theta_1 k_{t1} - \theta_2 k_{t1} + T - \mathrm{sgn}(\frac{d\theta_2}{dt}) f_1 W_1 D_1 \qquad 2.15$$

$$M_3 \frac{d^2 x_3}{dt^2} = x_1 \cos^2\phi \, k_m + \frac{dx_1}{dt} \cos^2\phi \, c_m$$

$$+ y_1 \cos\phi \sin\phi \, k_m + \frac{dy_1}{dt} \cos\phi \sin\phi \, c_m$$

$$+ \theta_1 r_1 \cos\phi \, k_m + \frac{d\theta_1}{dt} r_1 \cos\phi \, c_m$$

$$- x_3 (\cos^2\phi \, k_m + k_{xx2} B_{33} + \frac{1}{A_{33}})$$

$$
\begin{aligned}
& - \frac{dx_3}{dt} (\cos^2\phi \; c_m + c_{xx2} \; B_{33}) \\
& - y_3 \; (\cos\phi \; \sin\phi \; k_m + k_{xy2} \; B_{33}) \\
& - \frac{dy_3}{dt} (\cos\phi \; \sin\phi \; c_m + c_{xy2} \; B_{33}) \\
& + \theta_3 \; r_3 \; \cos\phi \; k_m + \frac{d\theta_3}{dt} r_3 \; \cos\phi \; c_m \\
& + x_4 \; (k_{xx2} \; B_{34} + \frac{1}{A_{34}}) + \frac{dx_4}{dt} c_{xx2} \; B_{34} \\
& + y_4 \; k_{xy2} \; B_{34} + \frac{dy_4}{dt} c_{xy2} \; B_{34} \\
& + M_3 \; e_3 \; \omega_2{}^2 \cos(\omega_2 \; t + \phi_3) \\
& + \frac{A_{44}}{A_{34}} M_4 \; e_4 \; \omega_2{}^2 \cos(\omega_2 \; t + \phi_4)
\end{aligned}
\tag{2.16}
$$

$$
\begin{aligned}
M_3 \frac{d^2 y_3}{dt^2} = {} & x_1 \cos\phi \; \sin\phi \; k_m + \frac{dx_1}{dt} \cos\phi \; \sin\phi \; c_m \\
& + y_1 \sin^2\phi \; k_m + \frac{dy_1}{dt} \sin^2\phi \; c_m \\
& + \theta_1 \; r_1 \; \sin\phi \; k_m + \frac{d\theta_1}{dt} r_1 \; \sin\phi \; c_m \\
& - x_3 \; (\cos\phi \; \sin\phi \; k_m + k_{yx2} \; B_{33}) \\
& - \frac{dx_3}{dt} (\cos\phi \; \sin\phi \; c_m + c_{yx2} \; B_{33})
\end{aligned}
$$

$$- y_3 \left(\sin^2\phi \; k_m + k_{yy2} \; B_{33} + \frac{1}{A_{33}}\right)$$

$$+ \theta_3 \; r_3 \; \sin\phi \; k_m + \frac{d\theta_3}{dt} \; r_3 \; \sin\phi \; c_m$$

$$+ x_4 \; k_{yx2} \; B_{34} + \frac{dx_4}{dt} \; c_{yx2} \; B_{34}$$

$$+ y_4 \left(k_{yy2} \; B_{34} + \frac{1}{A_{34}}\right) + \frac{dy_4}{dt} \; c_{yy2} \; B_{34}$$

$$+ M_3 \; e_3 \; \omega_2^2 \; \sin(\omega_2 \; t + \phi_3)$$

$$+ \frac{A_{44}}{A_{34}} \; M_4 \; e_4 \; \omega_2^2 \; \sin(\omega_2 \; t + \phi_4) \qquad 2.17$$

$$J_3 \; \frac{d^2\theta_3}{dt^2} = - x_1 \; r_3 \; \cos\phi \; k_m - \frac{dx_1}{dt} \; r_3 \; \cos\phi \; c_m$$

$$- y_1 \; r_3 \; \sin\phi \; k_m - \frac{dy_1}{dt} \; r_3 \; \sin\phi \; c_m$$

$$- \theta_1 \; r_1 \; r_3 \; k_m - \frac{d\theta_1}{dt} \; r_1 \; r_3 \; c_m$$

$$+ x_3 \; r_3 \; \cos\phi \; k_m + \frac{dx_3}{dt} \; r_3 \; \cos\phi \; c_m$$

$$+ y_3 \; r_3 \; \sin\phi \; k_m + \frac{dy_3}{dt} \; r_3 \; \sin\phi \; c_m$$

$$+ \theta_3 \; (r_3^2 \; km + k_{t2}) + \frac{d\theta_3}{dt} \; r_3^2 \; cm$$

$$+ \theta_4 \; k_{t2} - \frac{T}{GR} - \text{sgn}\left(\frac{d\theta_3}{dt}\right) \; f_2 \; W_3 \; D_3 \qquad 2.18$$

$$M_4 \frac{d^2 x_4}{dt^2} = x_3 \left(k_{xx2} B_{34} + \frac{1}{A34}\right) + \frac{dx_3}{dt} c_{xx2} B_{34}$$

$$+ y_3 k_{yx2} B_{34} + \frac{dy_3}{dt} c_{yx2} B_{34}$$

$$- x_4 \left(k_{xx2} B_{44} + \frac{1}{A_{44}}\right) - \frac{dx_4}{dt} c_{xx2} B_{44}$$

$$- y_4 k_{xy2} B_{44} - \frac{dy_4}{dt} c_{xy2} B_{44}$$

$$+ m_4 e_4 \omega_2^2 \cos(\omega_2 t + \phi_4)$$

$$+ \frac{A_{33}}{A_{34}} M_3 e_3 \omega_2^2 \cos(\omega_2 t + \phi_3) \qquad 2.19$$

$$M_4 \frac{d^2 y_4}{dt^2} = x_3 k_{yx2} B_{34} + \frac{dx_3}{dt} c_{yx2} B_{34}$$

$$+ y_3 \left(k_{yy2} B_{34} + \frac{1}{A_{34}}\right) + \frac{dy_3}{dt} c_{yy2} B_{34}$$

$$- x_4 k_{yx2} B_{44} - \frac{dx_4}{dt} c_{yx2} B_{44}$$

$$- y_4 \left(k_{yy2} B_{44} + \frac{1}{A44}\right) - \frac{dy_4}{dt} c_{yy2} B_{44}$$

$$+ M_4 e_4 \omega_2^2 \sin(\omega_2 t + \phi_4)$$

$$+ \frac{A_{33}}{A_{34}} M_3 e_3 \omega_2^2 \sin(\omega_2 t + \phi_3) \qquad 2.20$$

$$J_4 \frac{d^2 \theta_4}{dt^2} = \theta_3 k_{t2} - \theta_4 k_{t2} + \frac{T}{GR} - \mathrm{sgn}\left(\frac{d\theta_4}{dt}\right) f_2 W_3 D_3 \qquad 2.21$$

Analytical Program Structure

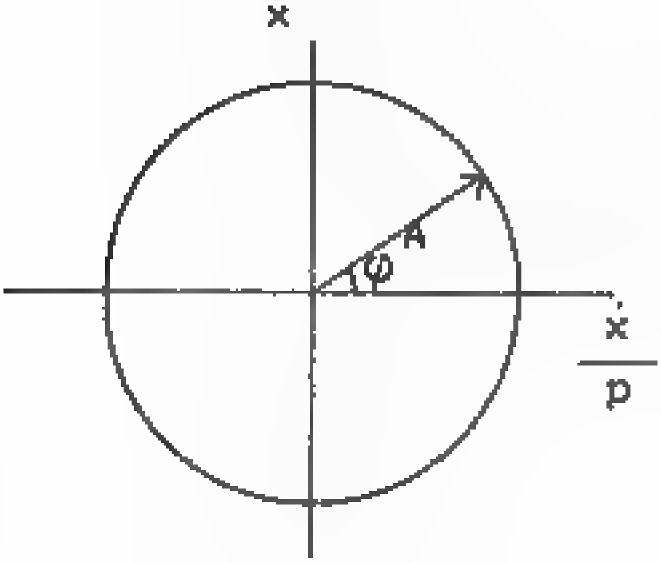
The equations obtained in the previous section are programmed with the structure detailed in Fig. 2.9 and Fig. 2.10. The main technique used to solve twelve coupled, time varying differential equations is the phase plane method which is a versatile tool for the analysis of this type of system.

Twelve phase planes are monitored simultaneously to analyze the response of the system shown in Fig. 2.2. The primary advantage of the technique over other numerical technique is that integration is based on the incremental angle in the phase plane rather than time. This allows very efficient integration of the equation of motion for systems with a wide band response spectrum.

The phase plane for undamped free vibration of a single degree of freedom system is shown in Fig. 2.7. The motion has an amplitude A, natural frequency ω_n, and the phase angle φ. The displacement and the velocity can be reduced to a single equation representing a circle.

$$x^2 + (\frac{dx}{dt} / \omega_n)^2 = A^2 \qquad 2.22$$

This method can be applied to complicated systems by rewriting the equations of motion as in Fig. 2.8. The integration procedure is shown in Fig. 2.9, and Fig. 2.10. Details can be found in the previous works by Seireg et al [29, 50].



$$m\ddot{x} + k\,x = 0$$

$$x = A\,\sin(p\Delta t + \varphi)$$

$$\dot{x} = A\,p\,\cos(p\Delta t + \varphi)$$

$$x^2 + \left(\frac{\dot{x}}{p}\right)^2 = A^2$$

Fig. 2 7 Phase Plane for Undamped Free Vibration

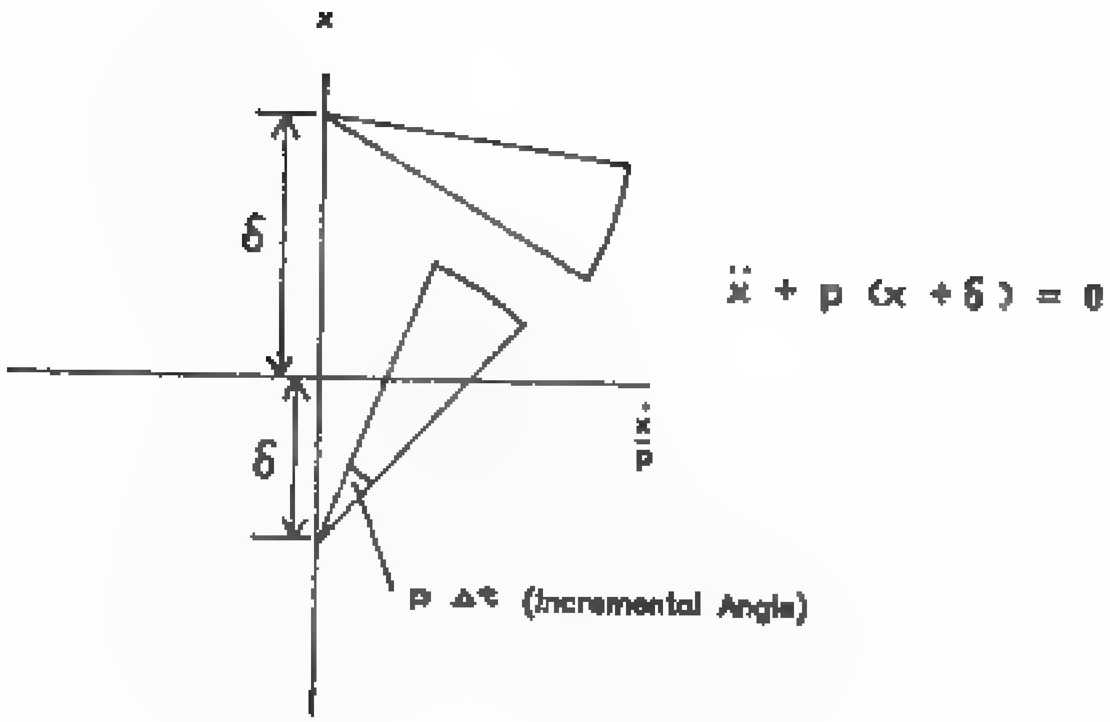


Fig. 2.8 Phase Plane δ Method

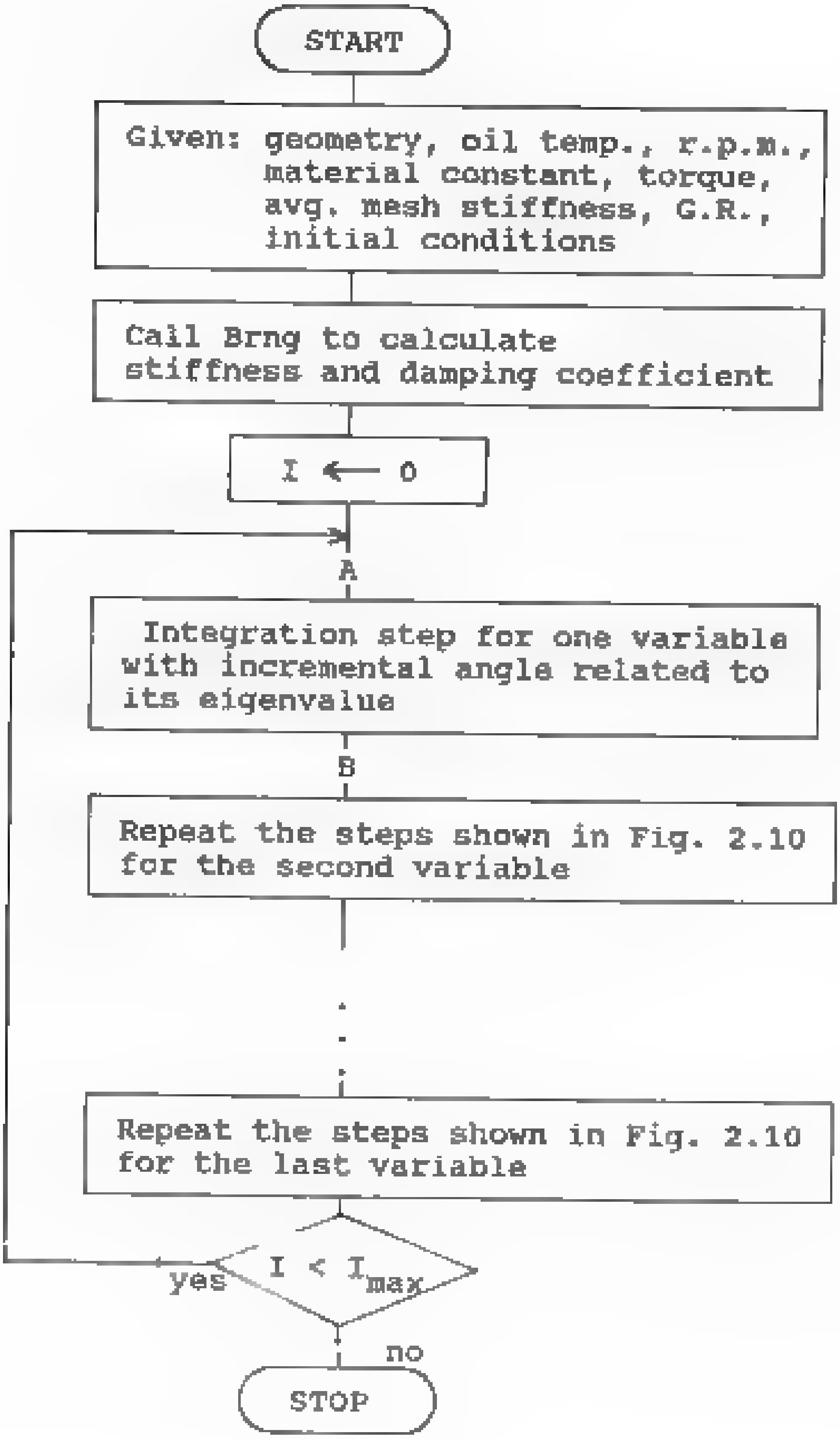

START
Given: geometry, oil temp., r.p.m., material constant, torque, avg. mesh stiffness, G.R., initial conditions
Call Brng to calculate stiffness and damping coefficient
I ← 0
A
Integration step for one variable with incremental angle related to its eigenvalue
B
Repeat the steps shown in Fig. 2.10 for the second variable
Repeat the steps shown in Fig. 2.10 for the last variable
I < I_max
yes
no
STOP

Fig. 2.9 Structure of Program to Solve Equations of Motion Using Phase Plane δ Method

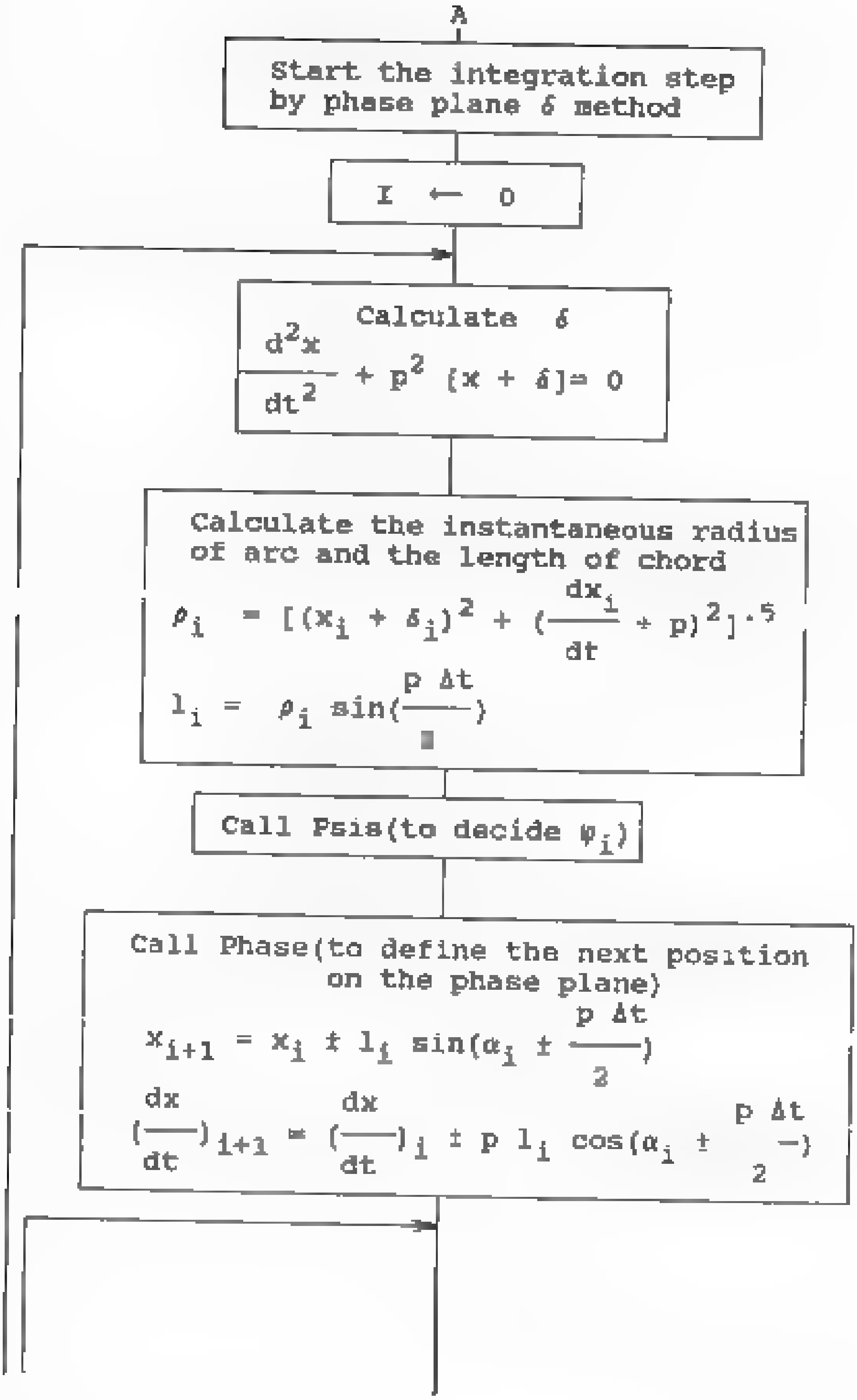

A
Start the integration step by phase plane δ method
I ← 0
Calculate δ
$\frac{d^2x}{dt^2} + p^2 (x + \delta) = 0$
Calculate the instantaneous radius of arc and the length of chord
$\rho_i = [(x_i + \delta_i)^2 + (\frac{dx_i}{dt} \div p)^2]^{.5}$
$l_i = \rho_i \sin(\frac{p\,\Delta t}{\text{[illegible]}})$
Call Psis(to decide ψ_i)
Call Phase(to define the next position on the phase plane)
$x_{i+1} = x_i \pm l_i \sin(\alpha_i \pm \frac{p\,\Delta t}{2})$
$(\frac{dx}{dt})_{i+1} = (\frac{dx}{dt})_i \pm p\, l_i \cos(\alpha_i \pm \frac{p\,\Delta t}{2})$

Fig. 2.10 Details of Integration Step in Fig. 2.9

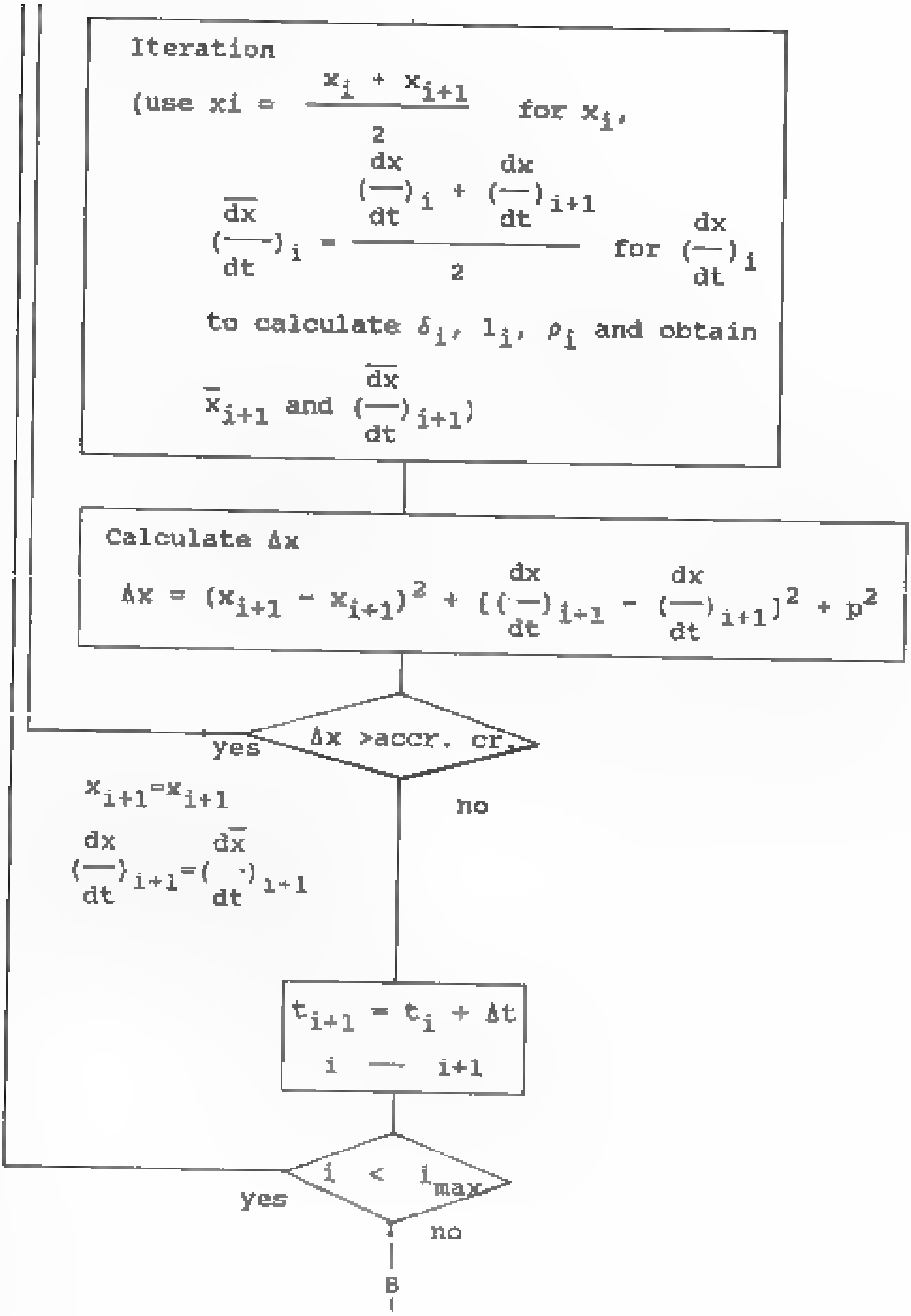

Iteration
(use $xi = \frac{x_i + x_{i+1}}{2}$ for x_i,
$\overline{(\frac{dx}{dt})}_i = \frac{(\frac{dx}{dt})_i + (\frac{dx}{dt})_{i+1}}{2}$ for $(\frac{dx}{dt})_i$
to calculate δ_i, l_i, ρ_i and obtain
$\bar{x}_{i+1}$ and $(\frac{\overline{dx}}{dt})_{i+1}$)
Calculate Δx
$\Delta x = (x_{i+1} - x_{i+1})^2 + [(\frac{dx}{dt})_{i+1} - (\frac{dx}{dt})_{i+1}]^2 + p^2$
Δx >accr. cr.
yes
$x_{i+1} = x_{i+1}$
$(\frac{dx}{dt})_{i+1} = (\frac{d\overline{x}}{dt})_{i+1}$
no
$t_{i+1} = t_i + \Delta t$
$i \longrightarrow i+1$
$i < i_{max}$
yes
no
B

Fig. 2.10 continued

Uncoupled Natural Frequencies and the Significant Parameters of the System

Since all the unstable situations are related to the natural frequencies of the system, it is helpful to calculate the uncoupled natural frequencies of individual modes for the isolated system to have a better insight into the coupled system. Geared systems, such as those investigated in this study, have many uncoupled modes as follows.

Torsion of Shaft

The natural frequency of the torsional vibration due to the torsional stiffness of shaft between rotor 1 and rotor 2 (or rotor 3 and rotor 4) is

$$\omega_n = 1.414 \frac{G\ I_{z1}}{.5\ M_1\ r_1^{2}\ (L_2 + L_3)} \qquad 2.23$$

Pinion-Gear System

Assuming that the motion is confined along the line of action, the natural frequency of gear-pinion mesh (Fig. 2.4) can be written as

$$\omega_n = 1.414\ (\frac{k_o}{M_1})^{.5} \qquad 2.24$$

Lateral Vibration of Simple Supported Mass due to Shaft Bending

For this mode, assuming a centrally located mass, the natural frequency can be calculated as follows:

$$\omega_n = \left(\frac{k_b}{M_1}\right)^{.5} \qquad 2.25$$

where

$$k_b = \frac{48\ E\ I_1}{(L_2 + L_3)^2} \qquad 2.26$$

Lateral Vibration of Overhang Mass due to Shaft Bending

The natural frequency in this case can be expressed as

$$\omega_n = \frac{k_{bo}}{M_2} \qquad 2.27$$

where

$$k_{bo} = \frac{6\ E\ I_1}{2\ L_1{}^2\ (L_1 + L_2 + L_3)\ 64} \qquad 2.28$$

The system, investigated in this study, is assumed to be symmetrical for simplification.

CHAPTER 3
SYSTEM RESPONSE AND STABILITY CONSIDERATION

The system investigated in the reported study is assumed to be symmetrical in order to simplify the computation. The following assumptions are made: $M_1 = M_3$, $L_2 = L_3 = L_5 = L_6$. Several cases are investigated to study the effect of the design parameters on the system stability and the possibility of introducing additional instability to neutralize the adverse effect of an expected instability.

In geared systems, instabilities can occur in the form of the lateral as well as torsional oscillations. The whirling and the torsional motion interact at the points of tooth contact. Accordingly, the amplitude build-ups can possibly be made to counteract and vanish.

Unstable situations are selected from the regions which represent practical operating speeds, and each case is stabilized by an additional excitation and by proper selection of the design and operating parameters including the bearing geometry, oils, geometry of the gear, and the phase shift of the induced disturbances.

Although the time history of the deflection of tooth, d_t, and gear whirl, d_g, are selected to represent the torsional and the lateral dynamics of the system respectively, all the twelve main variables(x_1, x_2, .., y_1, ..., θ_4) are calculated simultaneously and are examined when needed. The values of d_t and d_g are computed as

$$d_t = r_1 (\theta_1 + \theta_3) \qquad 3.1$$

$$d_g = (x_1^2 + y_1^2)^{.5} \qquad 3.2$$

The following are the main system parameters used to investigate the system used in the reported study.

ϕ = 20 degree
$\omega_1 = 2 \pi N / 60$ rad/sec
$\omega_2 = \omega_1$ (GR) rad/sec
$\omega_m = \omega_2$ (# of teeth of pinion) rad/sec
D_1 = 2.5"
D_2 = 2.5"
d_b(bearing diameter) = D_1 + 2 c inch
l_b(bearing length) = 2.5"
$L_2 = L_3 = L_5 = L_6$ = 4.325"
$M_1 = M_3$ = .169 slug
r_1 = 7"
GR = 1
W_t = 1.5"
T = W_t (1000) lb-in
$r_3 = r_1$ / GR
$k_g = 1.2 (10)^6$ lb/in
E = 30 (10^6) lb/in^2
Poisson's ratio = .292

The Selection of Bearing Parameters

The parameters of the oil film bearing for different situations are calculated as follows. In the case of stable system conditions, any bearing parameters which do not create negative damping are acceptable. However, in the case of system instability, the bearing parameters can be selected to supply the necessary amount of positive damping to the system. The system parameters are used to calculate the bearing parameters for the two rotor system. In the reported study, SAE 20 oil is selected as lubricant whose relative viscosity, μ, is $1.36\mathrm{x}10^{-8}$ reyns. Oil inlet temperature is assumed to be 110° F, and iteration method [39] is used to calculate the temperature increase, and accordingly viscosities at those operating temperatures which are calculated from the relationship

$$\mu(T) = \mu \exp[1271.5/(T + 95)]$$

along with the Sommerfeld numbers. Then, the Sommerfeld numbers are used to calculate the stiffness and the damping coefficients for given systems in different operating conditions.

Table 3.1 shows radial clearance of the fluid film bearings for each operating speed which is selected in an iterative way to reach the maximum damping for given

Table 3.1 Selection of Oil Film Bearings

Speed (rpm)	Radial Clearance(inch)
4200	.002
23875	.004
35812	.005
47750	.006
95500	.006

conditions. The subprogram to calculate the stiffness and the damping coefficients is listed in the Appendix.

Two Rotor System with Torque Inputs

A two rotor system with simply supported shafts is considered in this section. Three cases which represent stable, resonant, and unstable operations [2] are considered here to investigate the effect of using additional torque excitations and oil film bearings to suppress the amplitude of the oscillation due to a fluctuating torque input.

The system used in this section is shown in Fig. 2.1. It has six degrees of freedom and is originally supported on ball bearings. The system parameters are as mentioned before.

Stable Conditions

In this section a case is selected where no build-up occurs in the system response. If the amplitude of the response is acceptable, no additional treatment is necessary. However, if the response amplitude is considered to be excessive, it can be reduced by an additional torque excitation and an appropriate design of the oil film bearing.

Case 1: Stable Response due to Torque Excitation on Rotor 1

The stable response of a two degree of freedom torsional system due to an oscillating torque input is calculated for the following conditions for the same system shown in Fig. 2.1.

$T_1 = W_t$ (1000) lb-in

ω_{ex} = 3750 rad/sec

ω_{no} = 5012 rad/sec

ω_r = 3750 rad/sec

Fig. 3.1 shows the deflection at the tooth contact. A beat can be seen in the response, but no build-up occurs in the motion. The beat is attributed to the difference between natural frequency and the excitation frequency. The frequency of oscillation is the same as the excitation frequency, and the beat frequency is approximately 1250 rad/sec which is equal to the difference between the natural frequency and the excitation frequency.

Case 2: The Effect of an Additional Torque Input on Rotor 3

An additional torque input which can give the same response amplitude is now imposed on the other gear. As a result, the amplitude of the oscillation becomes much smaller as shown in Fig. 3.2. Elimination of the large amplitude is essentially complete. Practically, there is no deflection of tooth spring. (The peak amplitude is reduced from 0.0004" to 0.0000006".)

In this case the amplitude of the second torque excitation is the same as the first torque excitation and the phase angle between the two is 180 degrees.

$$T_3 = - T_1$$

$$\omega_{ex} = 3750 \text{ rad/sec}$$

Case 3: The Effect of the Oil Film Bearings

Fluid film bearings with appropriate parameters (oil viscosity, clearance) is used to replace the original ball bearings. The damping in the bearing is utilized to reduce the amplitude of the oscillation of the system with the two torque inputs. The bearing length, in this study, is fixed with the ratio of $l_b/d_b = 1.0$. For this case, the following parameter values are selected.

$$c = 0.005''$$

$$\mu = 1.36/(10)^8 \text{ reyns}$$

As shown in Fig. 3.3, the amplitude of the tooth spring response is slightly reduced by the use of the oil film bearings. It can also be seen that the beat has disappeared as a result of the damping in the bearings.

It is interesting to note that as a result of using the oil film bearing, the system experiences a larger amount of whirling of the gear which was negligible when ball bearings were used. The amplitude of the whirl, however, is approximately 0.0013 % of the radial clearance and can therefore be tolerated (Fig. 3.4).

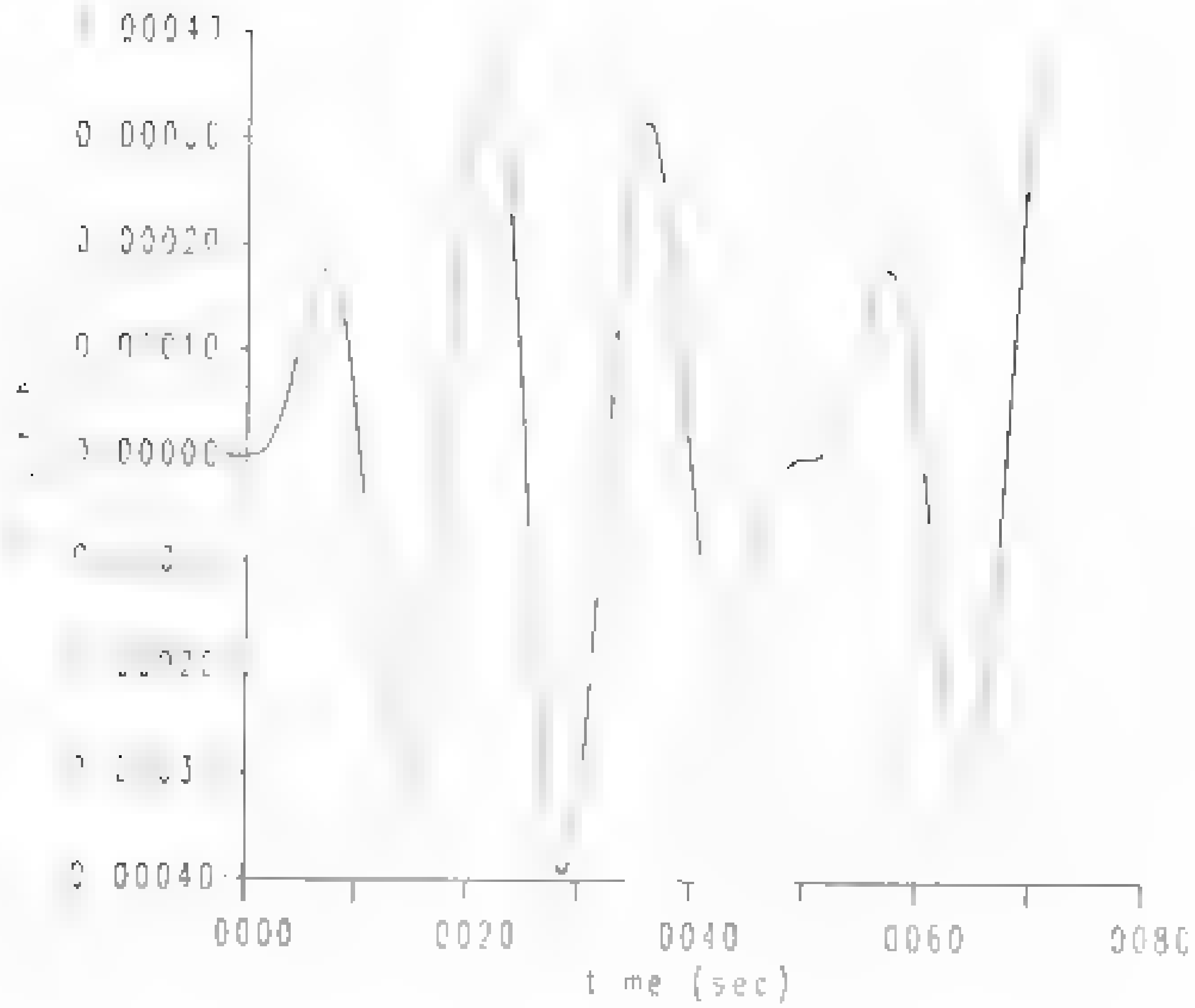


Fig. 3.1 Oscillatory Deflection of the Teeth at the Mesh due to a Torsional Excitation at Rotor 1 (Case 1) $\tau_{ex} = .0017$, $\tau_{no} = .00125$

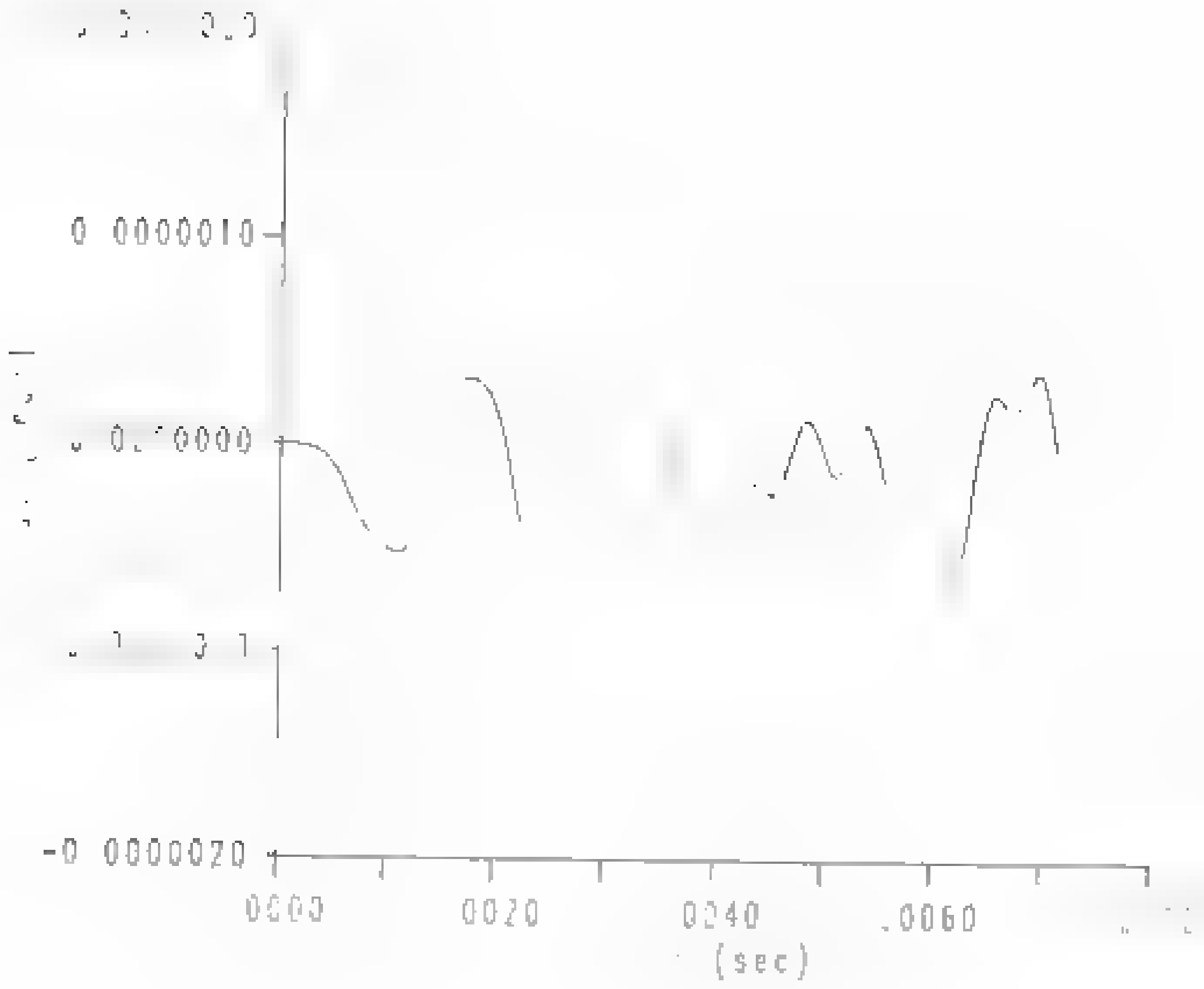


Fig. 3.2 Oscillatory Deflection of the Teeth at the Mesh due to Two Torques (Another Identical Torque with 180 Degree Phase Shift Imposed on the Other Rotor (Case 2)
τ_{ex} = .0017, τ_{no} = .00125

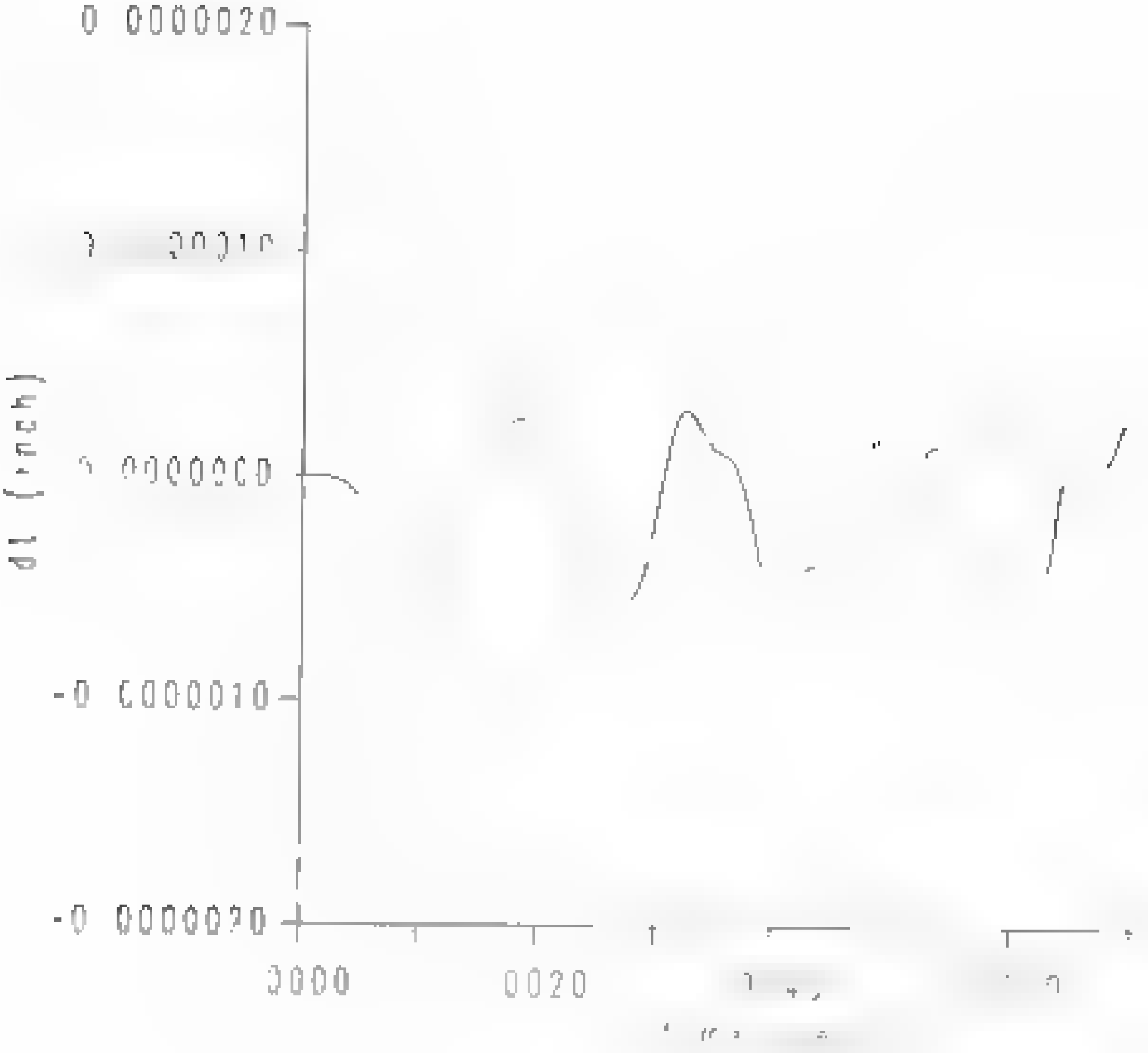


Fig. 3.3 Tooth Deflection at the Mesh due to Two Torques When the Shafts are Supported by Appropriate Oil Film Bearings (Case 3) $\tau_{ex} = .0017$, $\tau_{no} = .00125$

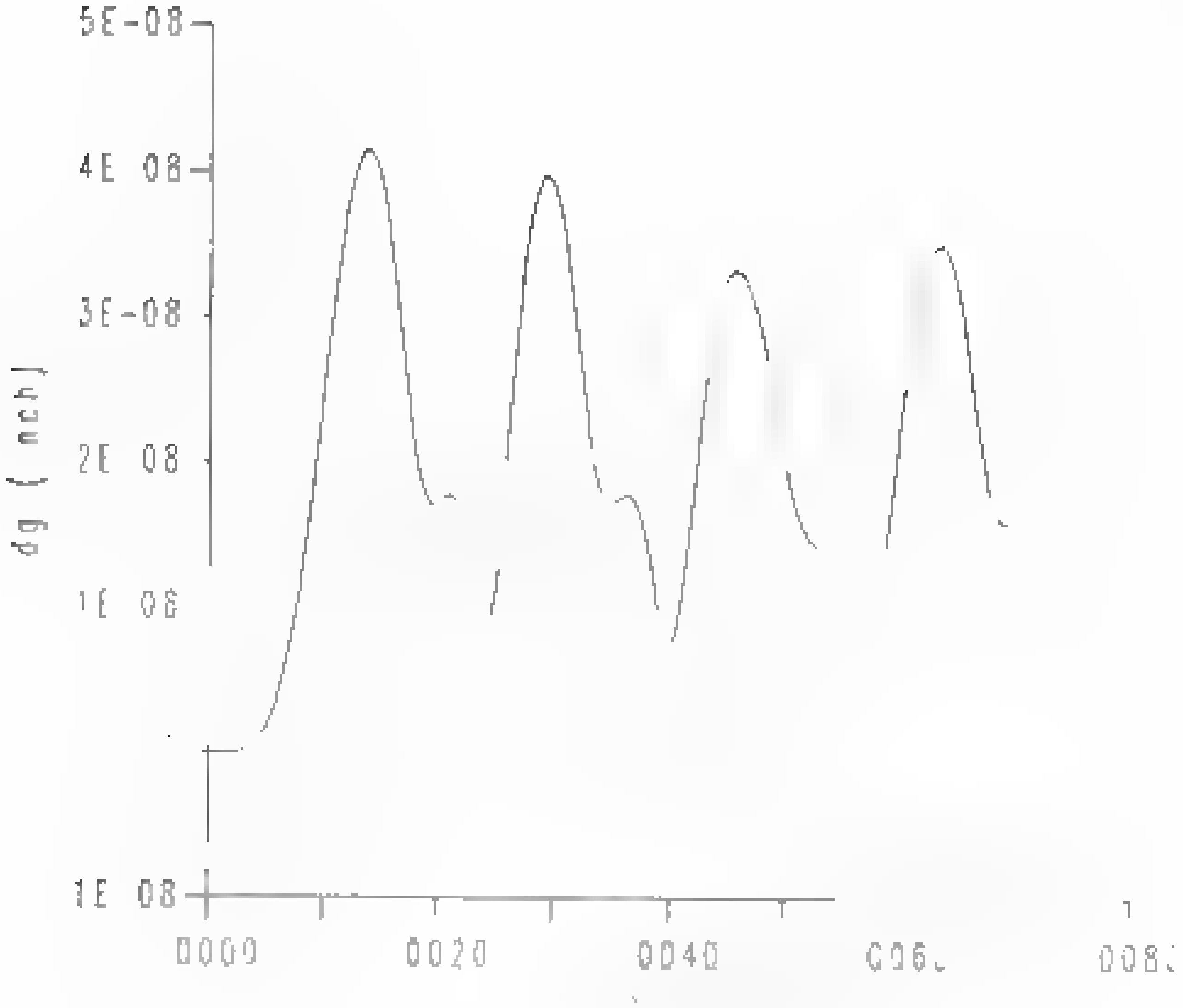


Fig. 3.4 Gear Whirl for Case 3
f_{ex} = .0017, f_{no} = .00125

Resonance conditions

The system experiences resonance when the exciting frequency is the same as the nominal natural frequency of the pinion-gear system.

$$\omega_{ex} = \omega_{no} = K_o \frac{(J_1+J_3)}{J_1 \; J_3} \qquad 3.3$$

Case 4: System Response at Resonance

The resonant response of the tooth spring displacement in the undamped condition due to an oscillatory torque excitation is expected to build up linearly with time.

Fig. 3.5 shows the response which is obtained for the following conditions.

$T_1 = W_t$ (1000) lb-in

ω_{ex} = 5000 rad/sec

ω_{no} = 5012 rad/sec

ω_r = 5000 rad/sec

The frequency of the response is the same as the nominal natural frequency of the pinion-gear system.

Case 5: The Effect of the Additional Torque Input

Another torque excitation at the resonant frequency is imposed on the other mass in order to neutralize the effect of the resonance resulting from the first excitation.

In this case the amount of the additional torque is same as the original one. A 180 degrees of phase shift of torque excitation was found to produce the best results.

$$T_3 = - T_1$$

$$\omega_{ex} = 5000 \text{ rad/sec}$$

The result which is plotted in Fig. 3.6 shows that the amplitude of the oscillation is suppressed drastically and the amplitudes of oscillation is approximately 0.1 % of the original value.

Case 6: The Effect of the Oil Film Bearings

Bearing length, oil viscosity, and clearance of the bearing can be selected in such a way to provide positive damping to the system with two torque excitations. For this case, the following parameter values are selected.

$$c = 0.006''$$

$$\mu = 1.36/(10)^8 \text{ reyns}$$

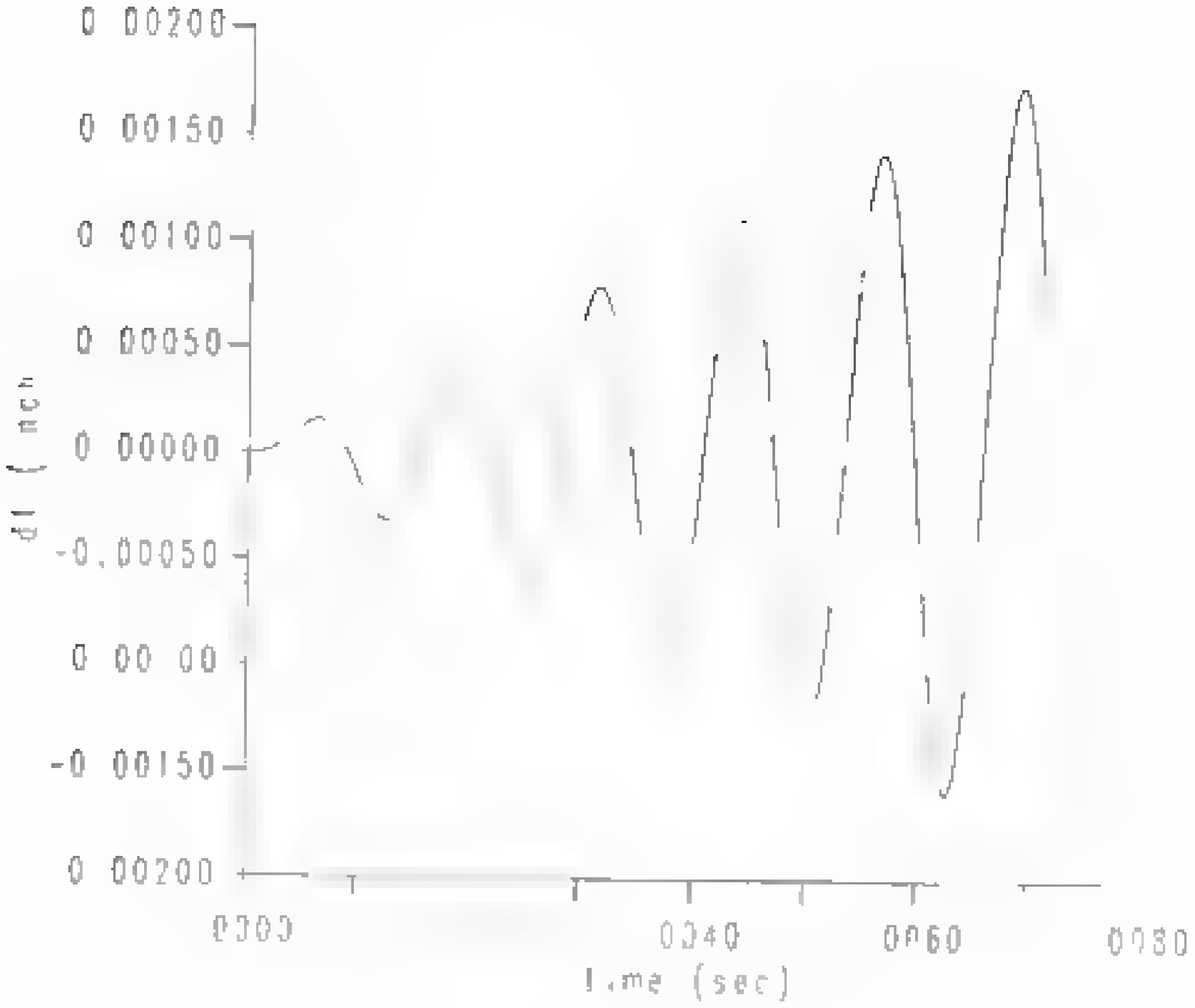


Fig. 3.5 Tooth Deflection at the Mesh for a Resonance due to a Torque Applied on Rotor 1 (Case 4) f_{ex} = .00125, f_{no} = .00125

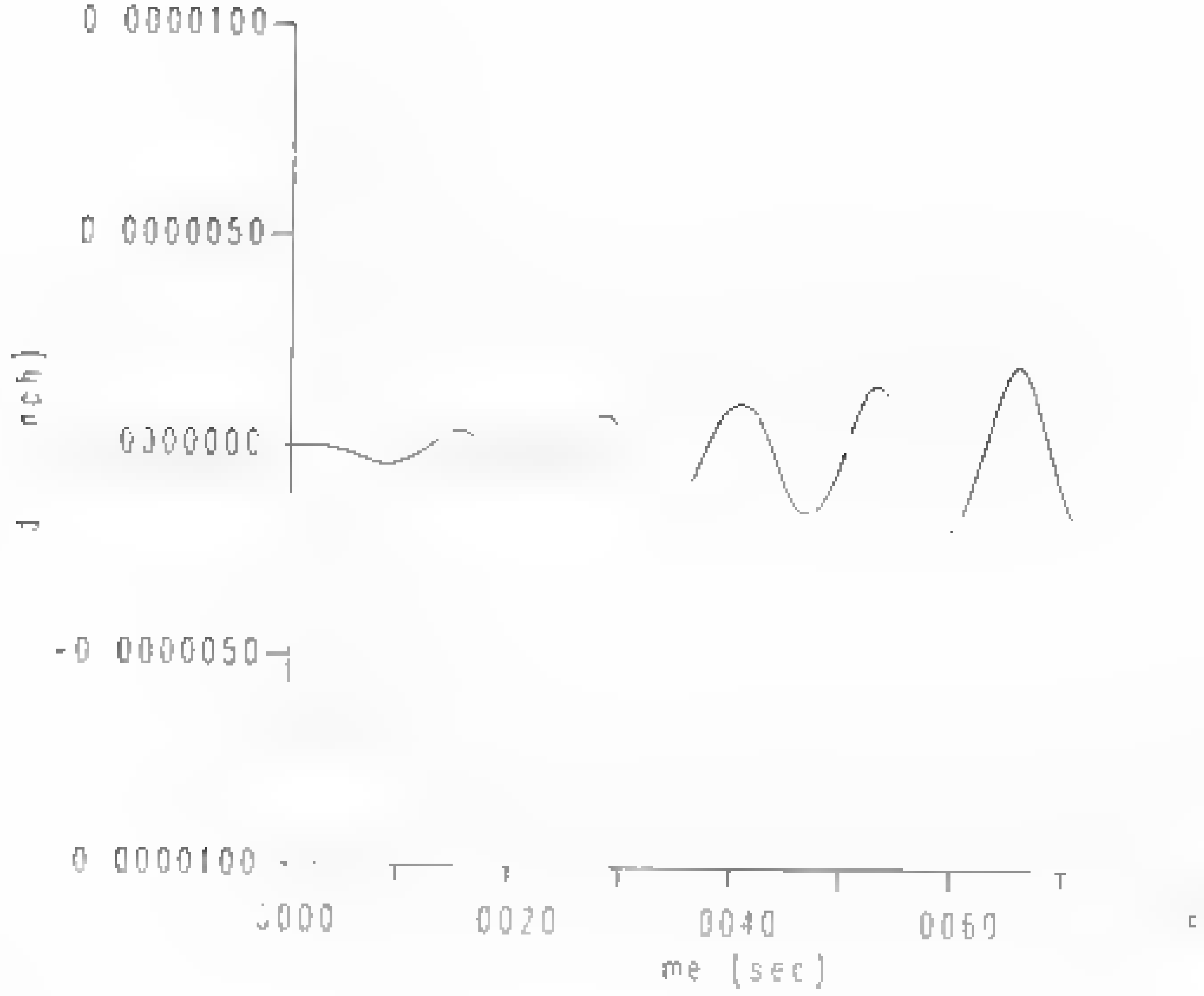


Fig. 3.6 Tooth Deflection at the Mesh due to Two Torque Applied on Two Rotors at Resonant Frequency (Case 5)
$\tau_{ex} = .00125$, $\tau_{no} = .00125$

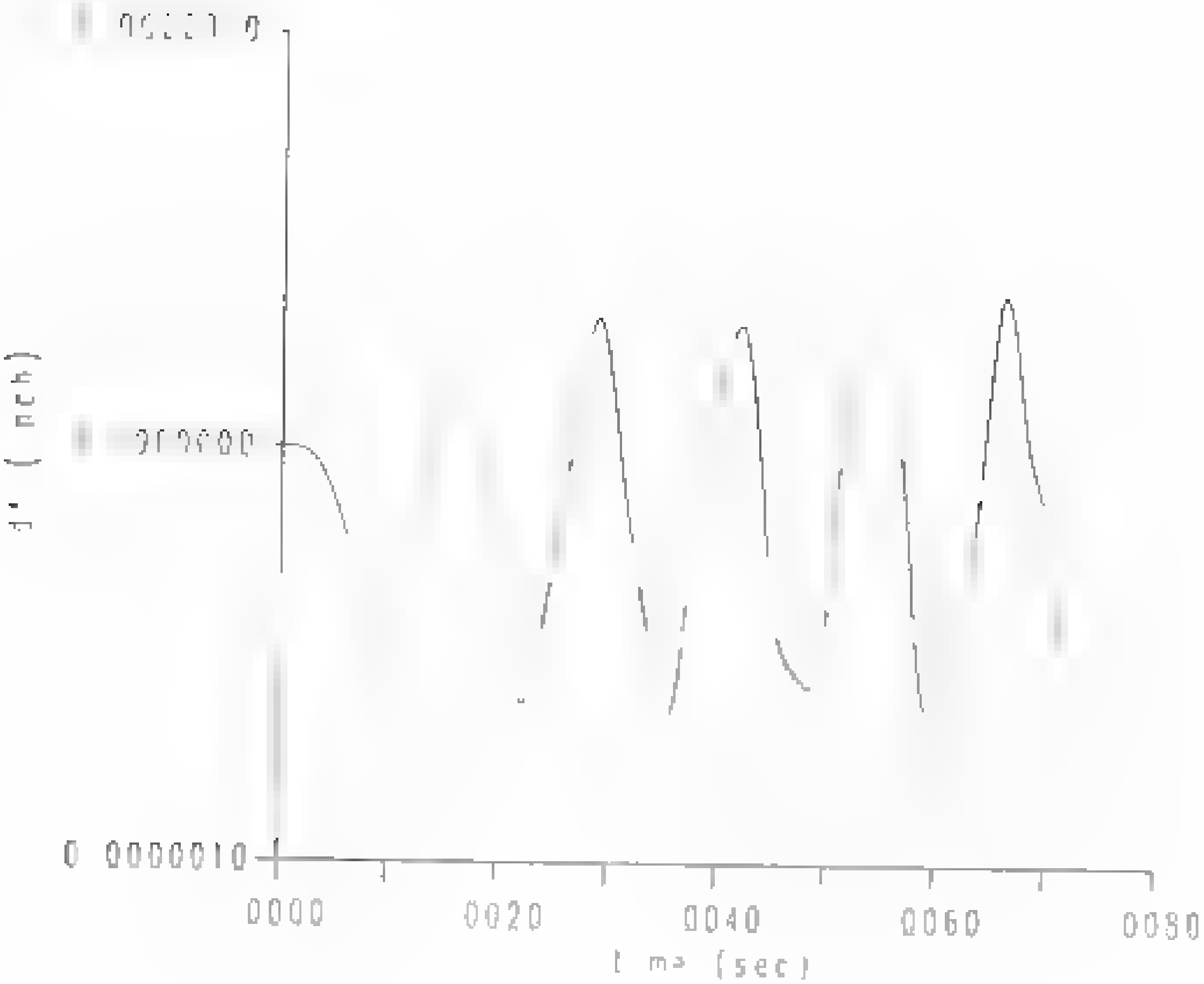


Fig. 3.7 Response due to Two Torque Excitations at Resonance with Appropriate Oil Film Bearing Support (Case 6)
τ_{ex} = .00125, τ_{no} = .00125

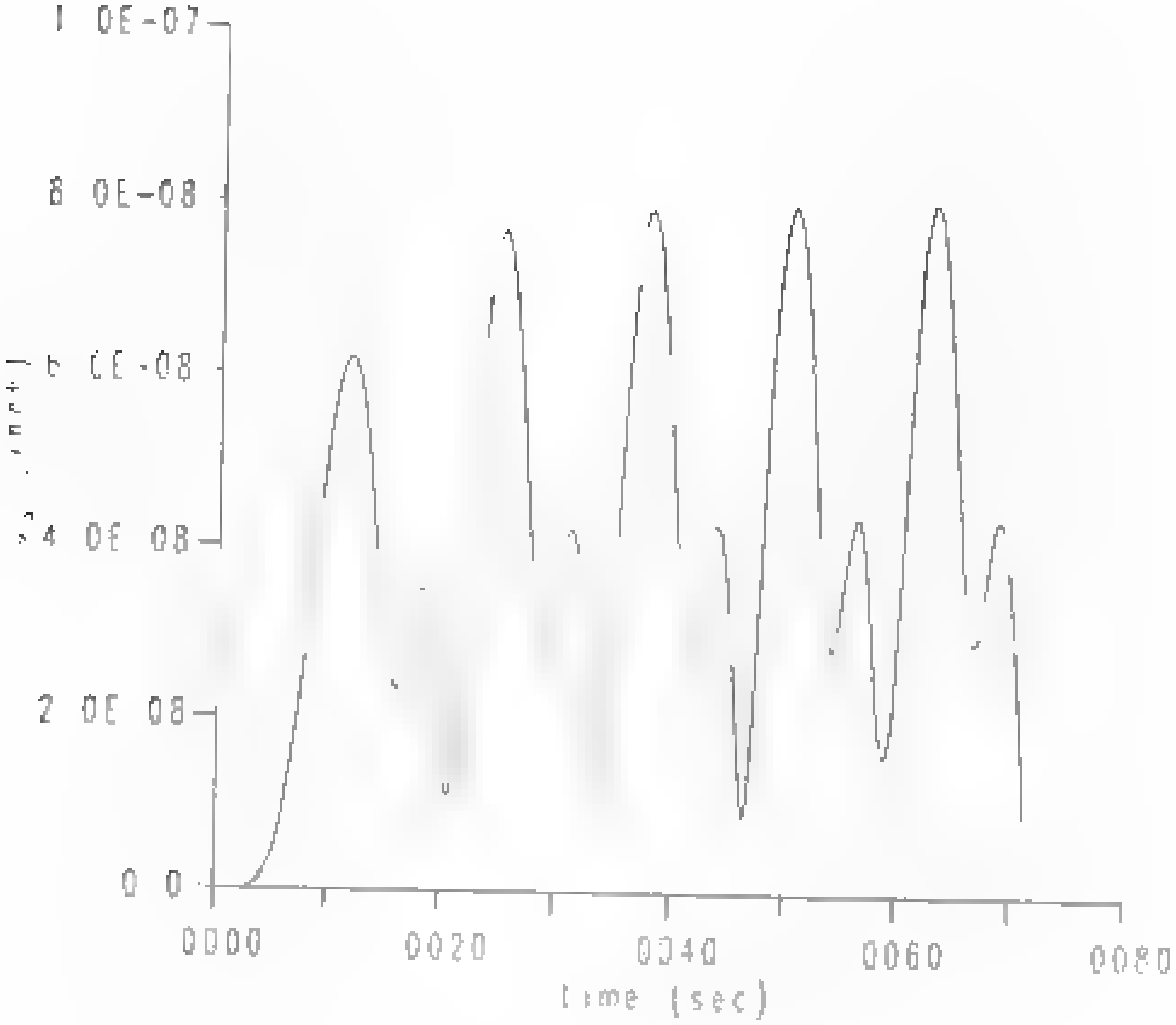


Fig. 3.8 Gear Whirl for Case 6
$\tau_{ex} = .00125$, $\tau_{no} = .00125$

As shown in Fig. 3.7, the response of the system has been suppressed more by the positive damping from the selected oil film bearing.

As in the stable case, oil film bearing causes gear whirl as shown in Fig. 3.8, which is approximately 0.0013 % of the clearance and is therefore of an acceptable magnitude.

Unstable System

An unstable situation, characterized by exponential build-up of the response amplitude, is expected under the following condition

$$\frac{\omega_{ex}}{\omega_{no}} = 2 \quad \frac{\omega_{ex}}{\omega_m} \qquad 3.4$$

This condition is selected based on the data given in Fig. 7(a) of the work by Benton and Seireg [2].

Case 7: Response of the Unstable System

The unstable response is obtained from condition which satisfies the relationship expressed in Eq. 3.4 between the excitation frequency, the nominal natural frequency of the pinion-gear system, and the mesh frequency.

$T_1 = W_t(1000)$ lb-in

$\omega_{ex} = 2500$ rad/sec.

$$\omega_{no} = 5012 \text{ rad/sec}$$

$$\omega_m = 10000 \text{ rad/sec}$$

$$\omega_r = 2500 \text{ rad/sec}$$

As shown in Fig. 3.9, torsional instability of the system is characterized by the exponential build-up pattern at the nominal natural frequency of the system which is twice the excitation frequency. This type of instability is called superharmonic resonance.

Case 8: The Effect of an Additional Torque

As in the resonance case, an additional torque input with identical system parameters and the proper phase shift can neutralize the effect of the instability due to an existing torque.

The response at the mesh with two torque inputs is shown in Fig. 3.10. The amplitude of the response has been reduced approximately to 0.1 % of the amplitude of the response due to one torque excitation by imposing the additional torque excitation with 180 degrees of phase shift.

$$T_3 = - T_1$$

$$\omega_{ex} = 2500 \text{ rad/sec}$$

Case 9: The Effect of the Oil Film Bearings

As in the example considered in case 6, the resonance amplitudes can be further reduced by using fluid film bearings to support the shafts.

The bearing parameters are selected as in previous cases to further reduce the torsional oscillations without inducing significant whirl. For this case, the following parameter values are selected.

$$c = 0.004''$$

$$\mu = 1.36/(10)^6 \text{ reyns}$$

As shown in Fig. 3.11, the amplitude of the response is suppressed and the build-up no longer exists.

The dominant frequency of the response is now the same as the excitation frequency, since the superharmonic resonance due to the negative damping created in the system is suppressed by the positive damping from the appropriately selected oil film bearing. The beat which appears in the response is due to the difference between the excitation frequency and the natural frequency. As in the previous cases, the oil film causes lateral gear oscillation as shown in Fig. 3.12. Again, the amplitude of gear whirl is less than 0.01 % of the radial clearance and is accordingly an acceptable magnitude.

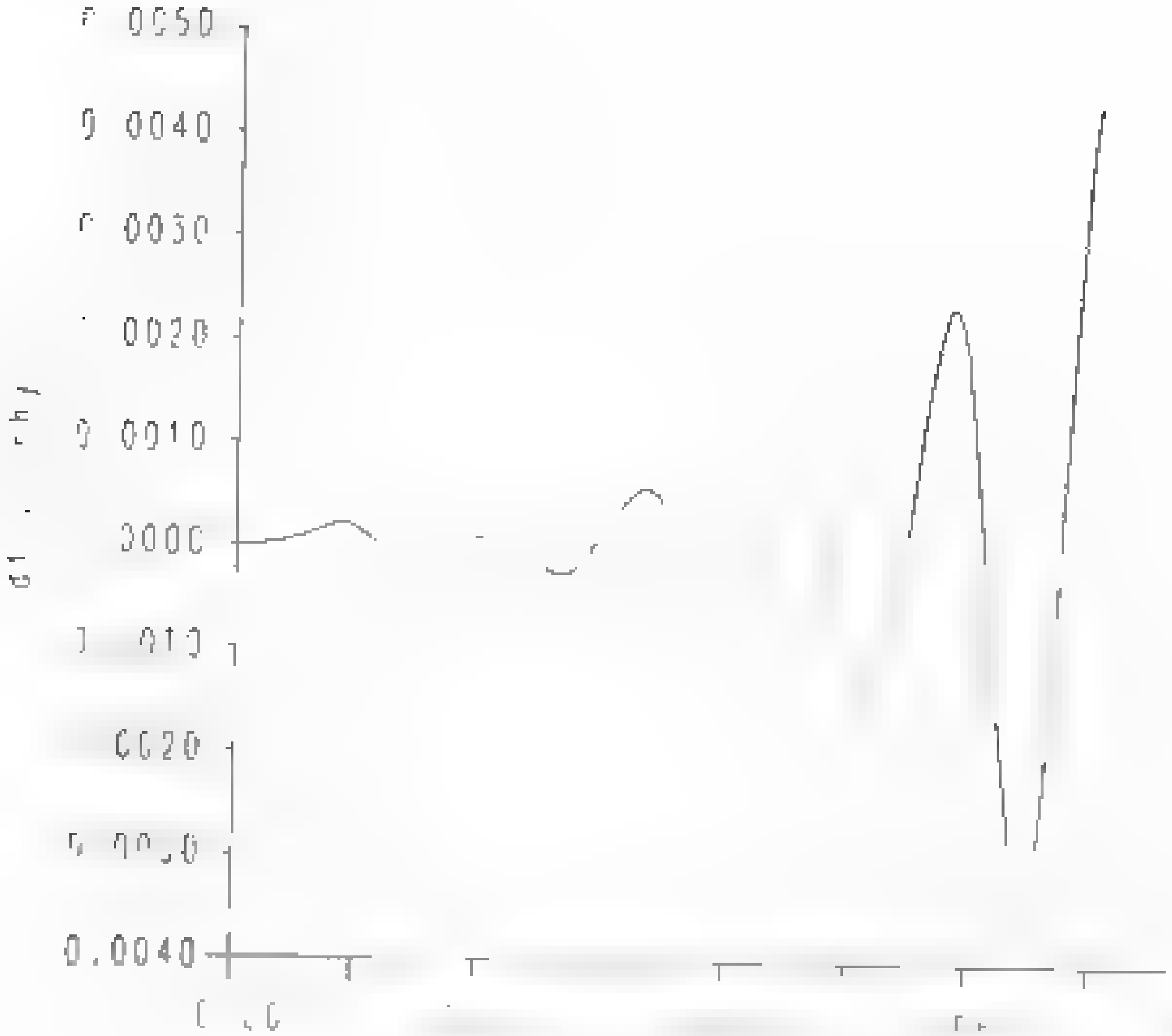


Fig. 3.9 System Response at the Tooth Mesh for a Superharmonic Instability (Case 7) r_{ex} = .0025, r_{no} = .00125

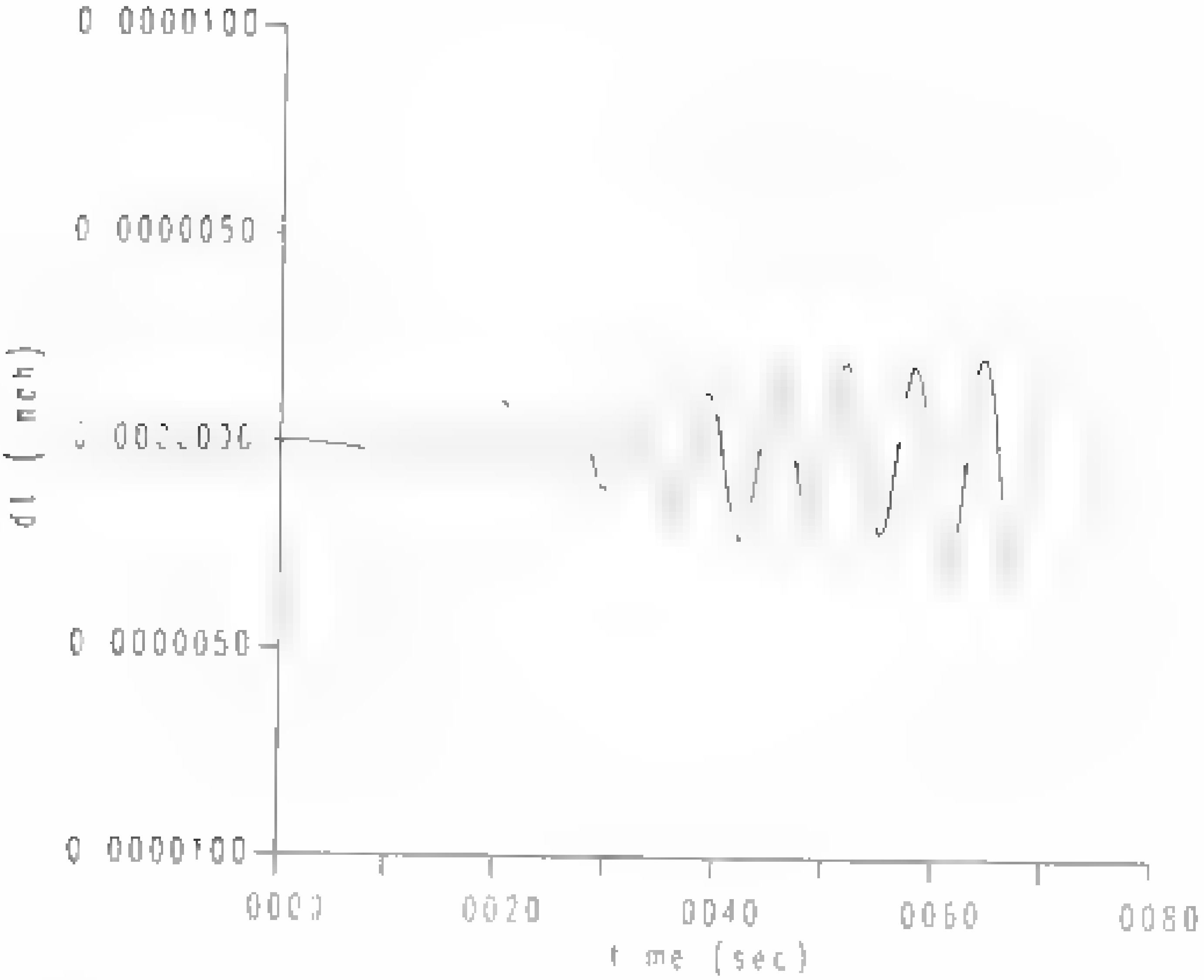


Fig. 3.10 Response due to Two Torque Excitations at Superharmonic Resonance (Case 8)
$\tau_{ex} = .0025$, $\tau_{no} = .00125$

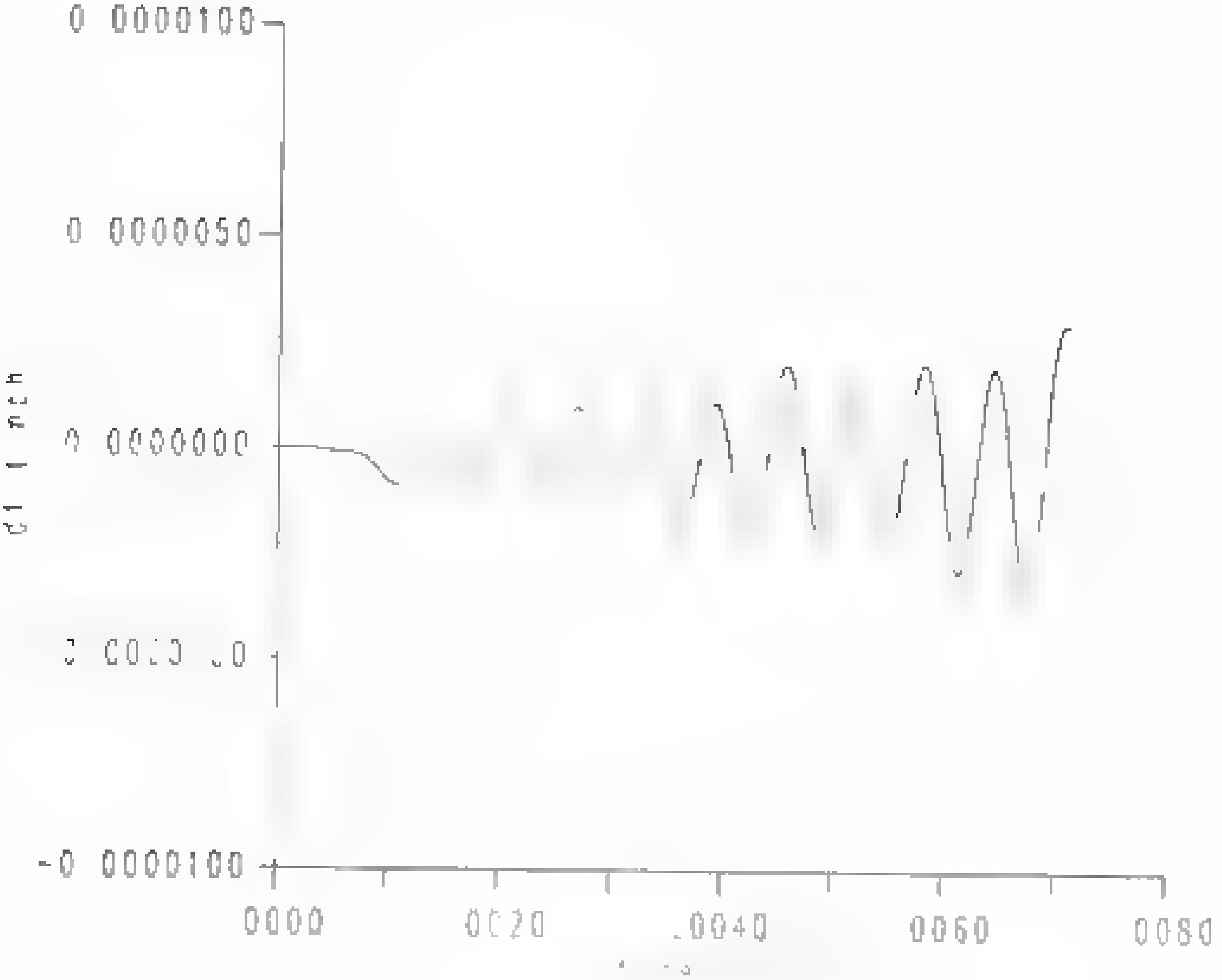


Fig. 3.11 Response due to Two Torque Excitations at Superharmonic Resonance with Appropriate Oil Film Bearing Support (Case 9) τ_{ex} = .0025, τ_{no} = .00125

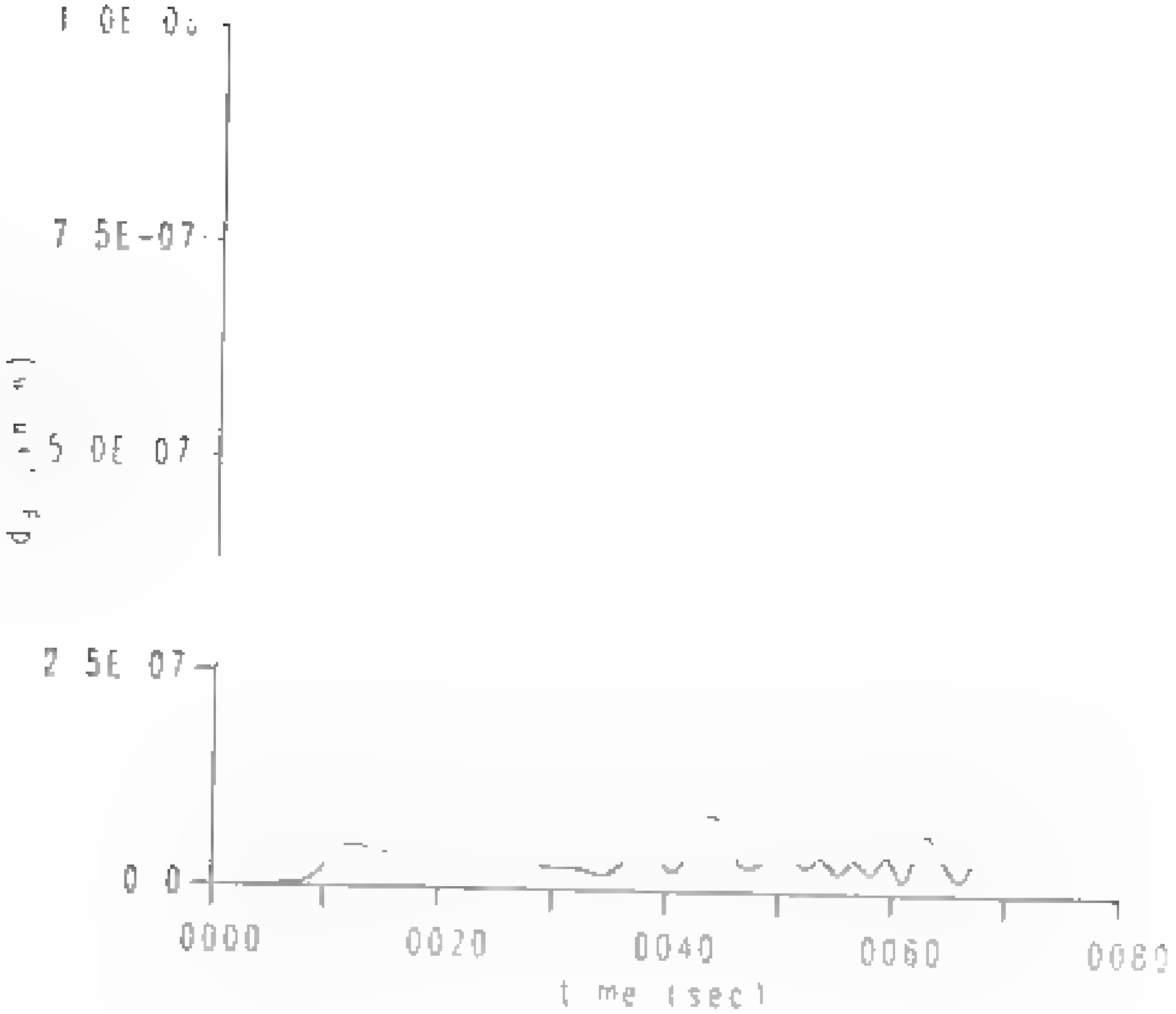


Fig. 3.12 Gear Whirl for Case 9
$\tau_{ex} = .0025$, $\tau_{no} = .00125$

It should be noted here that in this case the response of the system supported on the selected oil film bearings due to each torque input shows the same pattern of vibration. Two torque inputs, therefore, can neutralize the effect of each other with the proper amount of phase shift.

In other words, the large amplitude of oscillation due to a torque input can be avoided by another torque input with the proper amount of phase shift. The response of the system can be further suppressed by the damping provided by appropriate selection of the parameters of the oil film bearings.

Two Rotor System with Unbalance Excitations

The case of the coupled two rotor system subject to whirl is investigated in this section.

The effects of the gear coupling on the whirl of the system are a reduction of the whirl amplitude, a change in the resonant frequency, and the appearance of subharmonic as well as superharmonic resonances [22].

The possibility of suppressing the whirl due to an unbalance of one rotor by introducing an additional unbalance on the other rotor will be investigated for stable, resonant, and subharmonic resonant conditions.

The system is the same as that considered in section 3.1. The shafts are simply supported on ball bearings and the system has six degrees of freedom.

Stable Condition

A stable system without build-up of the whirl amplitude is investigated here. Even without build-up, the response of the system can reach unacceptable magnitudes if large amounts of unbalance exist. Either the amount of unbalance can be reduced or another unbalance can be imposed on the other gear to suppress the system response to the allowable limit. The latter case is discussed in this section in order to deal with high speed gear situations where perfect balance is difficult to achieve.

Case 10: Stable Response

The stable response of the coupled system due to an unbalance on one rotor is obtained for the same system illustrated in Fig. 2.1. The following conditions are considered.

$$e_1 = .00003/M_1 \text{ inch}$$

$$\omega_r = 3750 \text{ rad/sec}$$

$$\omega_y = 5029 \text{ rad/sec}$$

The whirl frequency can be calculated as follows.

$$\omega_w = [.5\ (\omega_x^2 + \omega_y^2)]^{.5} \qquad 3.5$$

where

$$\omega_y = \left(\frac{k_b}{M_1}\right)^{.5} \qquad 3.6$$

$$k_b = \frac{48\ E\ I_1}{(L_2 + L_3)^2} \qquad 2.24$$

The value of the equivalent natural frequency ω_w for the geared system is calculated according to the procedure given in the previous work by Seireg [20]. The lateral natural frequency along y direction, ω_y, is calculated without considering the gear constraint. The frequency ω_x is the natural frequency calculated in the same manner as ω_y with the exception of adding to the mass of the pinion an equivalent gear mass obtained by dividing the moment of inertia of the gear by the square of its pitch radius.

Since the system analyzed in this section is coupled at the mesh, an induced torsional motion is to be expected. The amplitude, however, of the torsional motion due to the unbalance is much less than that of gear whirl. Accordingly, the work will be focused on reducing the amplitude of the gear whirl.

The time history of the gear whirl is shown in Fig. 3.13. It can be seen that there is no build-up in this case as would be expected. The frequency of the response is the same as the rotation frequency of the shaft.

Case 11: Effect of an Additional Unbalance on the Other Rotor

An additional unbalance producing the same amount of eccentricity on the other rotor with 0 degree phase shift is introduced in the system to suppress the oscillation caused by the first unbalance. The result is illustrated in Fig. 3.14, which shows that the oscillation of the gear is essentially eliminated by introducing the additional unbalance on the other rotor.

$$e_3 = e_1$$

Case 12: The Effect of the Oil Film Bearings

The parameters of the oil film bearing considered in this analysis are as follows:

$$c = .005''$$
$$\mu = 1.36/(10)^8 \text{ reyns}$$

As shown in Fig. 3.15, the gear axis whirl is essentially eliminated when the considered oil film bearing is used to support the shaft. The coupling effect at the mesh causes torsional oscillations without the existence of any torsional excitation. The magnitude of this motion is practically negligible in this case as can be seen in Fig. 3.16.

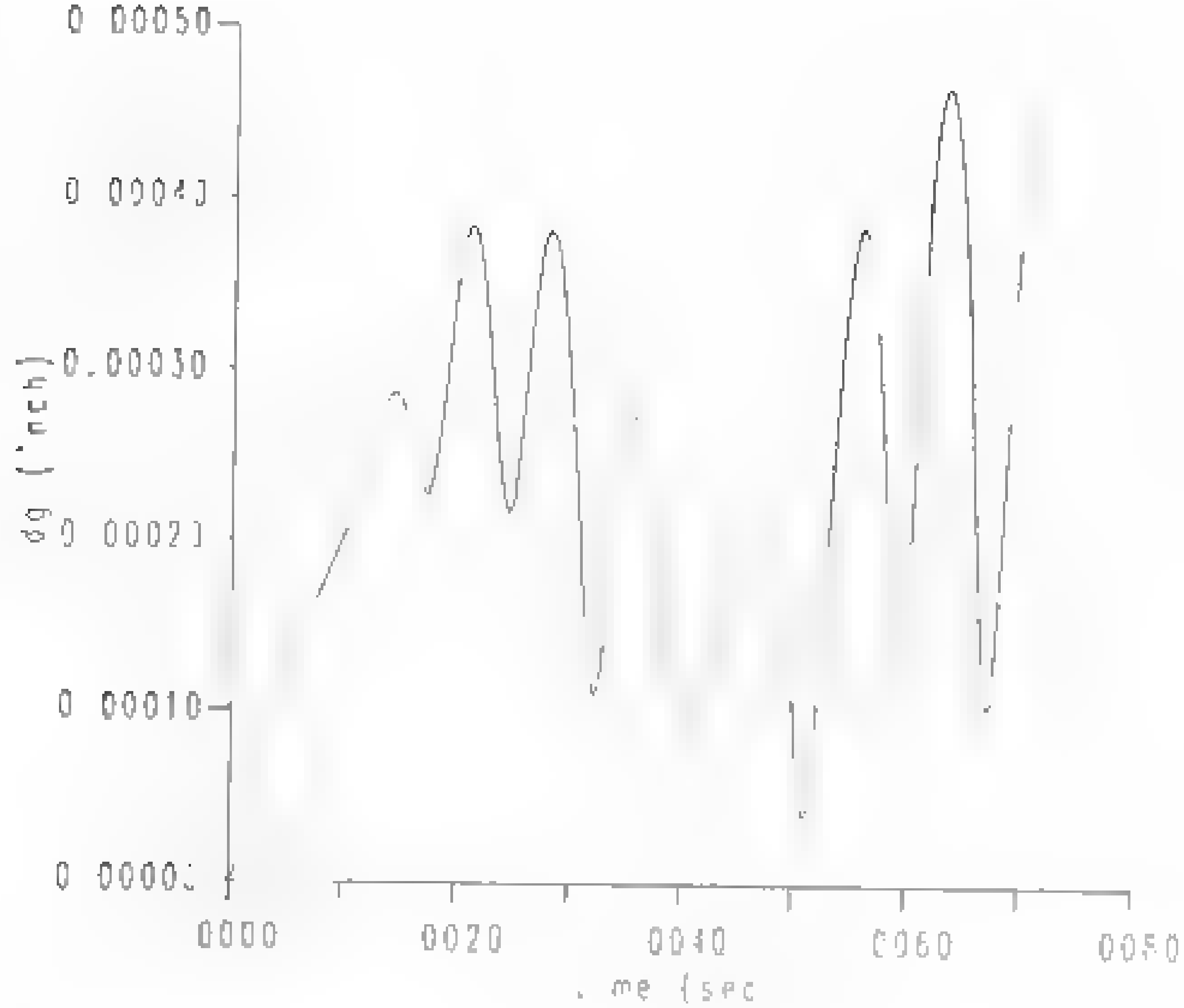


Fig 3.13 Gear Whirl for a Stable Condition with Unbalance on Rotor 1 (Case 10)
$\tau_r = .0017$, $\tau_y = .00125$

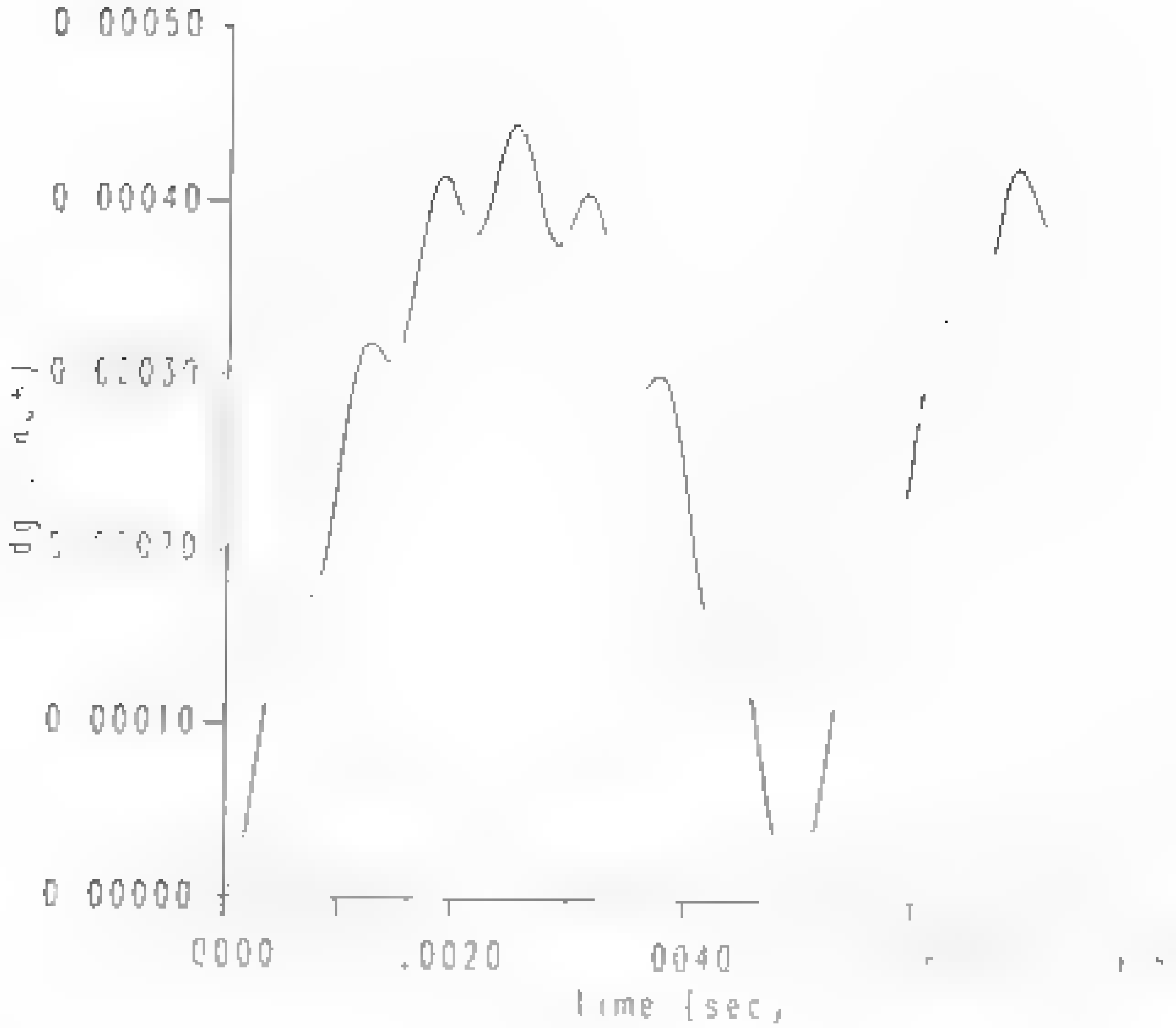


Fig. 3.14 Gear Whirl for a Stable Condition with Two In-phased Unbalances (Case 11) $\epsilon_r = .0017$, $\epsilon_y = .00125$

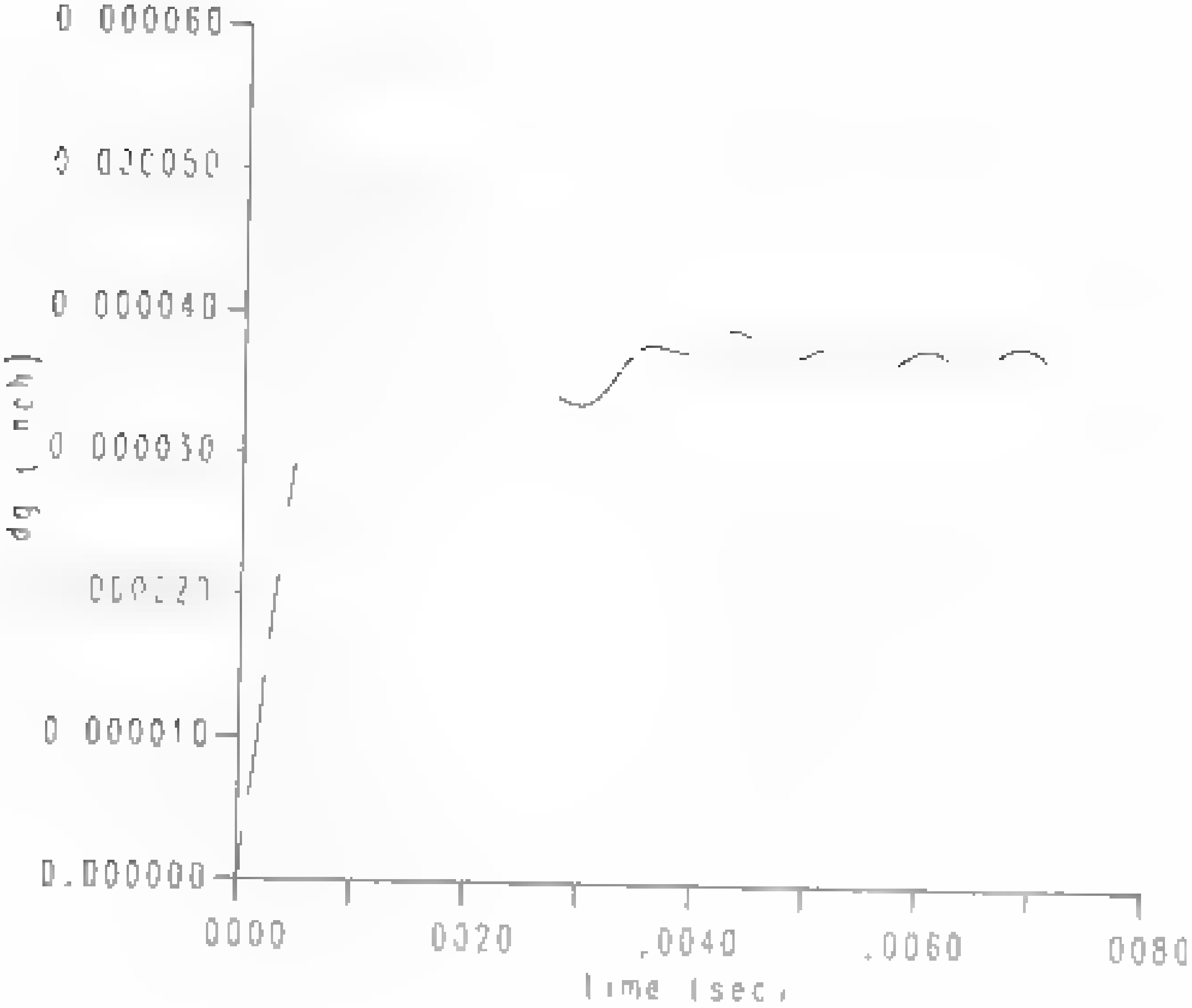


Fig. 3.15 Gear Whirl for the System with Two Unbalanced Rotors Supported on Fluid Film Bearings (Case 12)
r_r = .0017, r_y = .00125

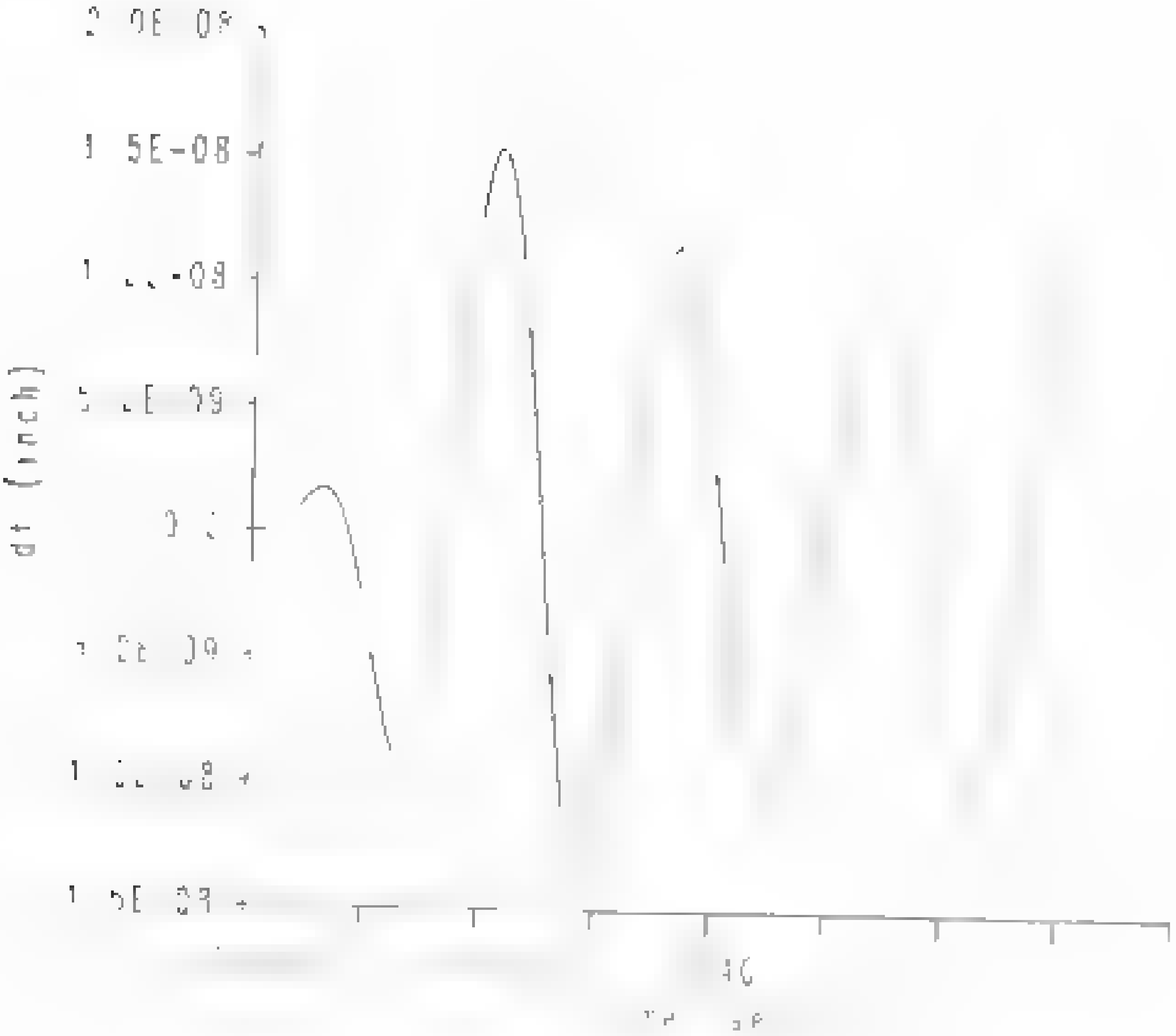


Fig. 3.16 Deflection of Gear Tooth for Case 12
$\tau_r = .0017,\ \tau_y = .00125$

System under Lateral Resonance Condition

Lateral resonance of the two rotor system can be induced when the rotating frequency of the shaft is the same as the equivalent lateral natural frequency. In this case the following value is used.

$$\omega_r = \omega_w$$

Case 13: Resonant Response with an Unbalance

A resonant case is investigated for the following conditions.

$$e_1 = .00003/M_1 \text{ inch}$$

$$\omega_r = 5000 \text{ rad/sec}$$

$$\omega_y = 5029 \text{ rad/sec}$$

The whirl motion of the gear axis in this case is plotted in Fig. 3.17 which illustrates the linear build up of the motion in this case with a frequency equal to the equivalent whirl frequency.

Case 14: Effect of an Additional Unbalance on the Other Rotor

Another whirl motion is generated by using an additional unbalance on the other rotor in order to suppress the amplitude of oscillation due to the original unbalance. The additional excitation on the other rotor due to the same amount of unbalance with 0 degree phase shift can reduce the deflection of gear tooth resulting from the first unbalance. The additional unbalance suppresses the high frequency oscillation due to the tooth contact induced by the first unbalance as shown in Fig. 3.18.

Case 15: Effect of the Oil Film Bearings

Fluid film bearings are used to support the shafts in this case. The parameters of the oil film bearing are selected in the same way as in the previous cases. The following parameter values are used as in the other unstable cases.

$$c = .006''$$

$$\mu = 1.36/(10)^8 \text{ reyns}$$

As shown in Fig. 3.19, the gear whirl is stabilized due to the positive damping provided by the selected oil film bearing. The deflection of gear tooth also shows acceptable magnitude (Fig. 3.20).

As in section 3.1, the large amplitude of the oscillation due to an unbalance can be suppressed by the additional unbalance on the other rotor with a phase shift of 0 degree. Further reduction of the whirl motion can be achieved utilizing the damping provided by the proper selection of the oil film bearing. It is interesting to note that, as in case of the gear system with fluctuating torque inputs, the additional excitation has its more significant effect on suppressing the amplitude of high frequency oscillation caused

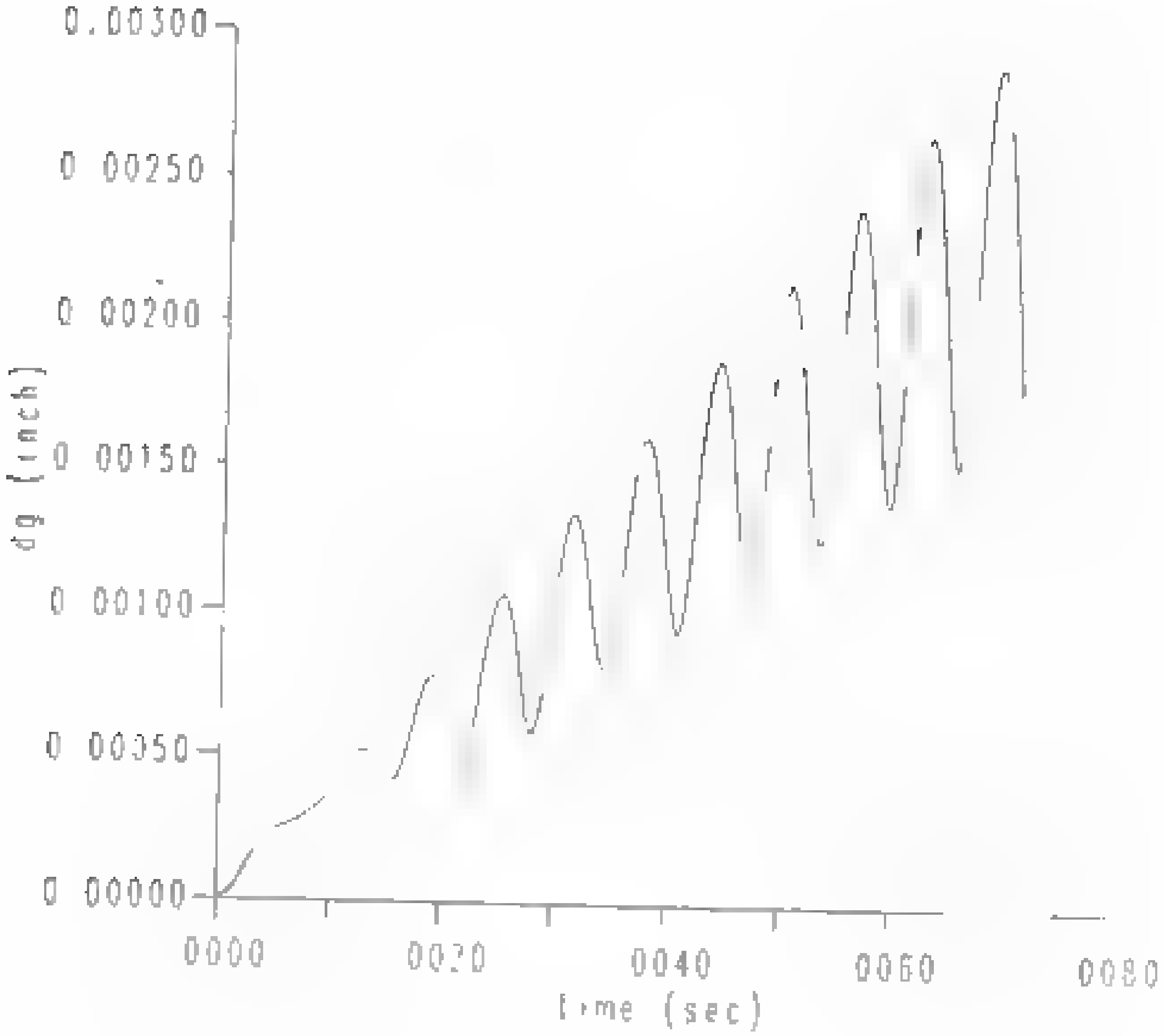


Fig. 3.17 Gear Whirl at Resonance (Case 13)
τ_x = .00125, τ_y = .00125

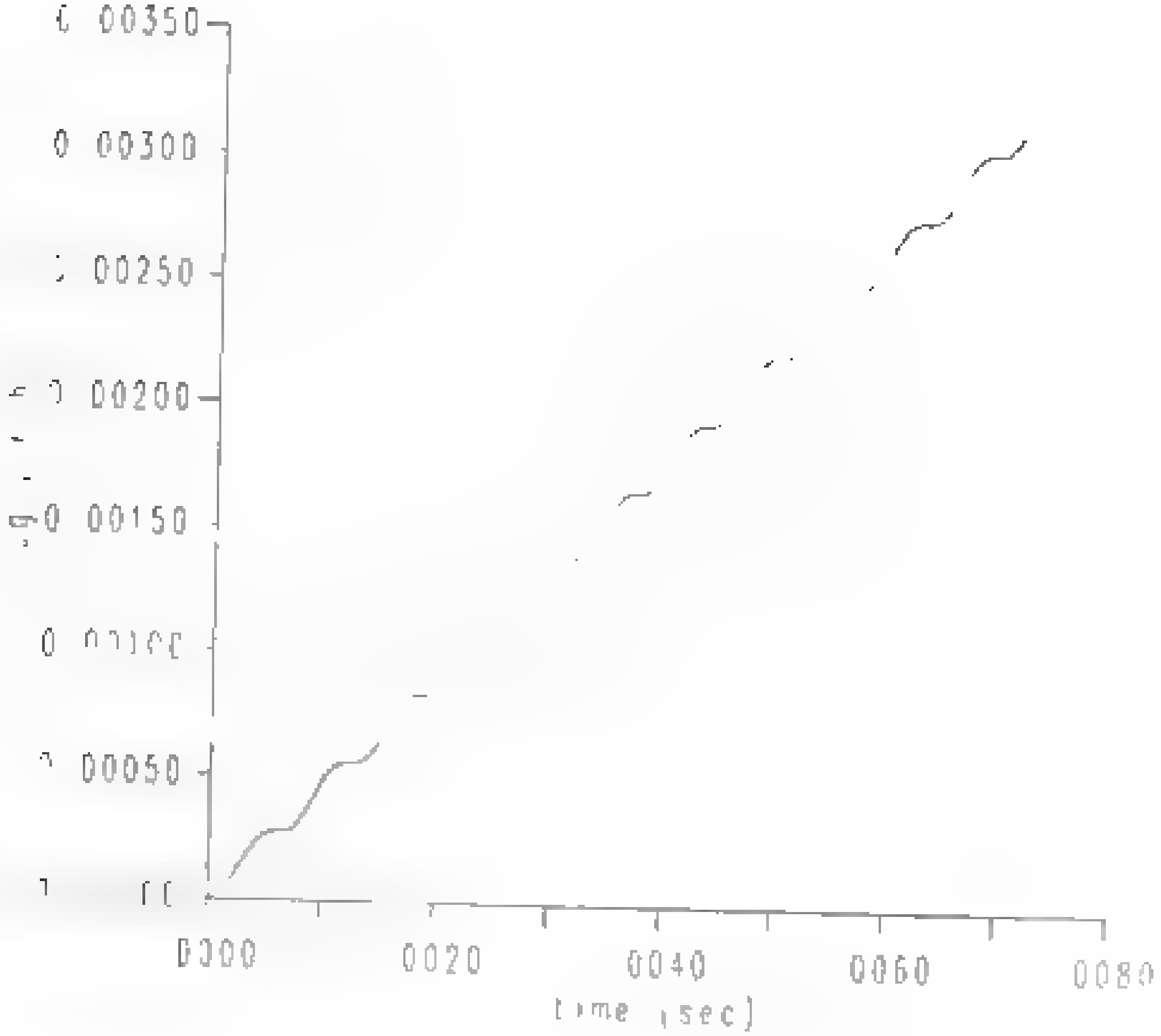


Fig. 3.18 Gear Whirl due to Two Unbalances on Two Rotors at Resonant Frequency (Case 14)
$r_r = .00125$, $r_y = .00125$

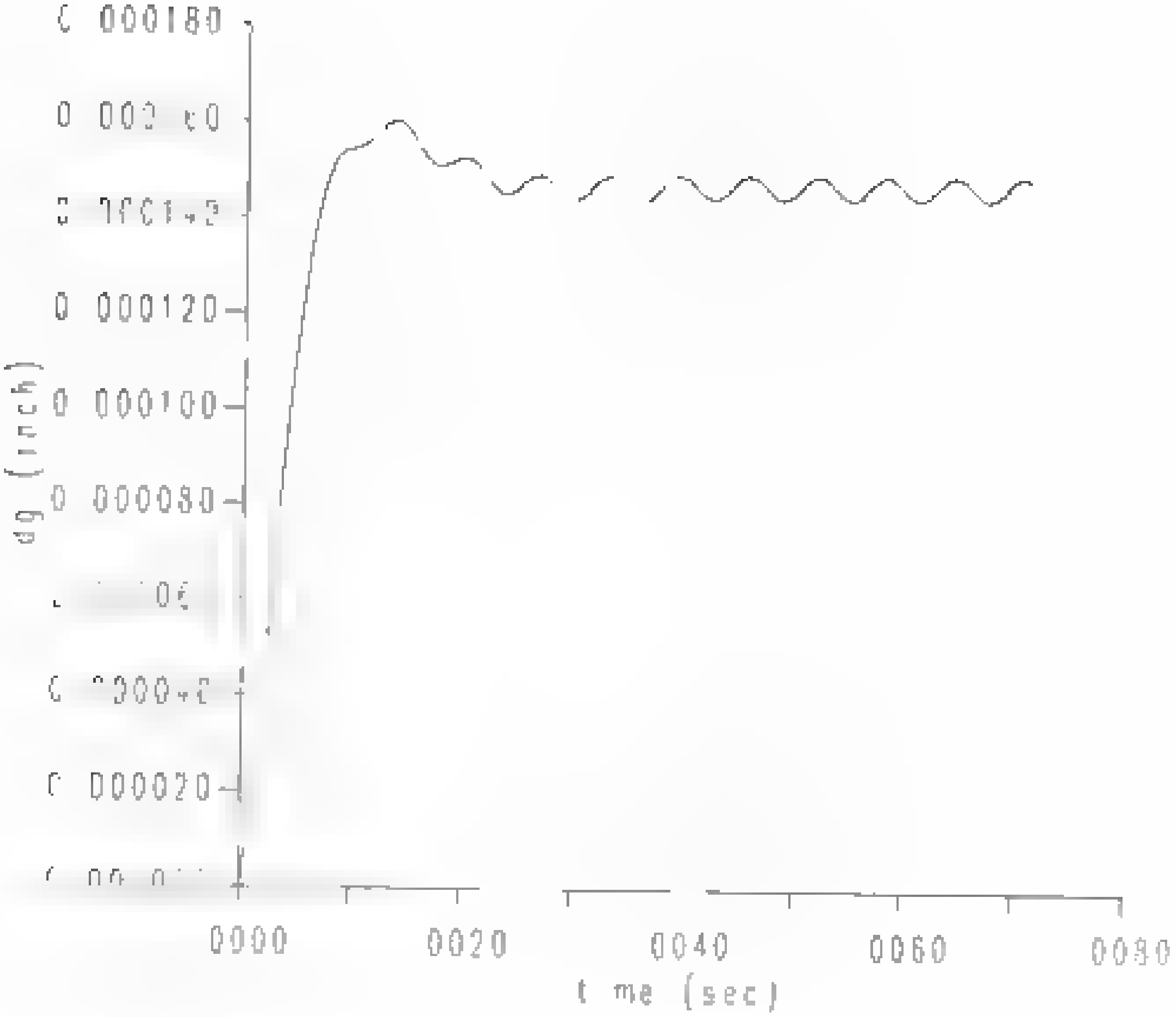


Fig. 3.19 Gear Whirl Subject to Two Unbalances with Appropriate Oil Film Bearings (Case 15, τ_r = .00125, τ_y = .00125

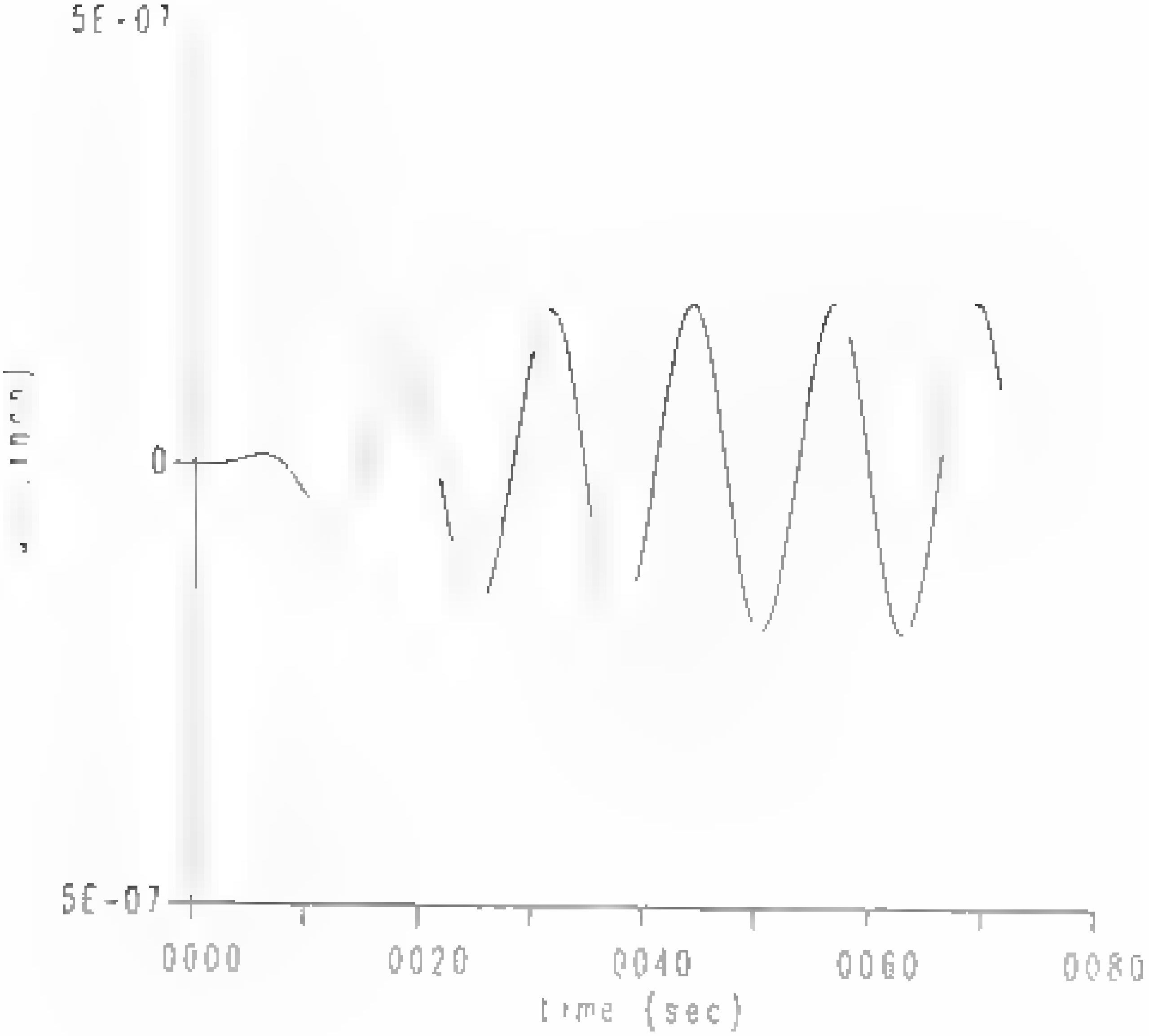


Fig. 3.20 Deflection of Gear Tooth for Case 15
τ_x = .00125, τ_y = .00125

by the tooth contact, and the damping due to oil film bearing works mainly on suppressing the build-ups.

System with Subharmonic Resonance

The instability of the two rotor system considered in this section is generated by inducing a subharmonic resonance situation as follows:

$$\omega_r = 2\ \omega_w$$

This condition is selected based on the data given in Fig. 8(a) of the previous work by Seireg [22].

Case 16: System Response with One Unbalance Excitation

The subharmonic resonant response is investigated for the following conditions.

$$e_1 = .00003/M_1 \text{ inch}$$

$$\omega_r = 10000 \text{ rad/sec}$$

$$\omega_y = 5029 \text{ rad/sec}$$

As shown in Fig. 3.21, the subharmonic resonance also shows build-up of the oscillatory motion.

Case 17. The Effect of an Additional Unbalance Excitation

As in the case of resonance, the large amplitude of the high frequency oscillation due to tooth contact excited by one unbalance can be suppressed by an additional unbalance excitation when the phase shift is 0 degree (Fig. 3.22).

Case 18: Effect of the Oil Film Bearings

The bearing parameters are selected as discussed previously. For this case, the following parameter values are selected.

$$c = .006"$$

$$\mu = 1.36/(10)^8 \text{ reyns}$$

As shown in Fig. 3.23, the gear whirl is stabilized, and the amplitude of deflection of gear tooth also attains an acceptable magnitude (Fig. 3.24).

In section 3.1, it has been demonstrated that the amplitude of torsional vibration due to an existing fluctuating torque can be suppressed by an additional torque excitation and the appropriate selection of the oil film bearing parameters. In this section, it has been shown that the lateral response due to an unbalance can be neutralized also by an additional unbalance imposed on the system and the appropriate selection of the oil film bearing parameters.

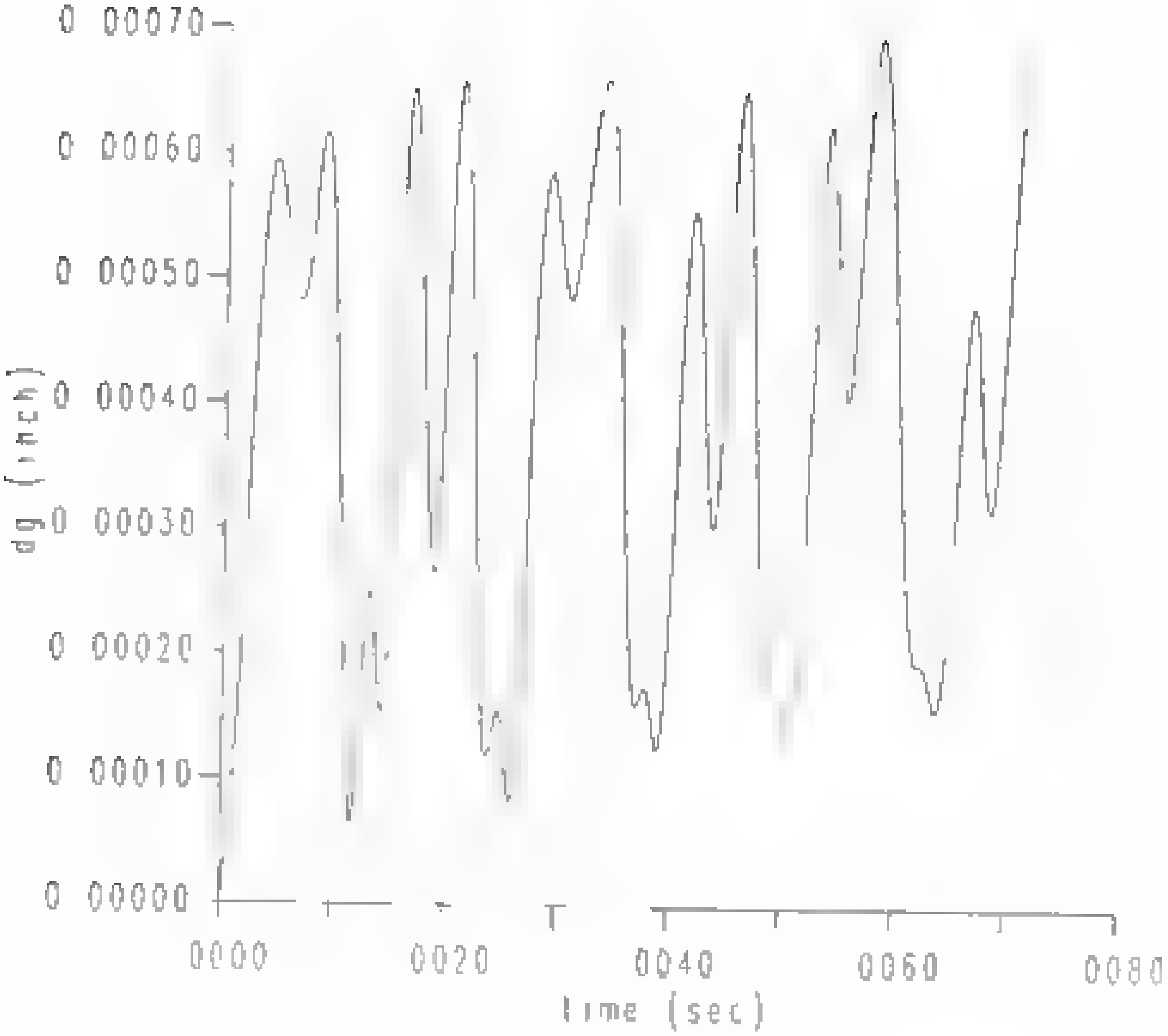


Fig 3.21 Gear Whirl at Subharmonic Resonance (Case 16)
τ_r = .00063, τ_y = .00125

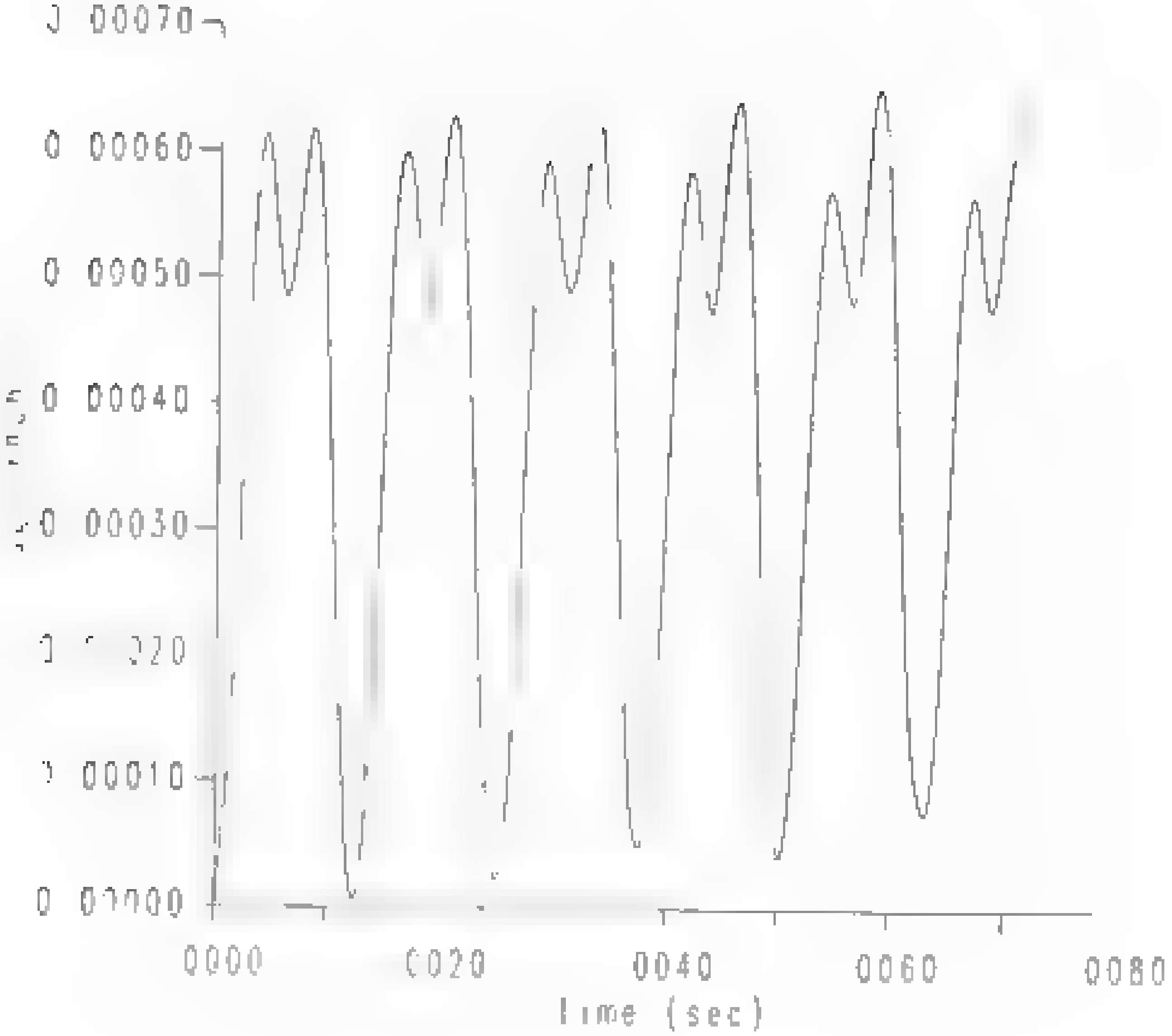


Fig. 3.22 Gear Whirl due to Two Unbalances on Two Rotors at Subharmonic Resonance (Case 17) $\tau_r = .00063$, $\tau_y = .00125$

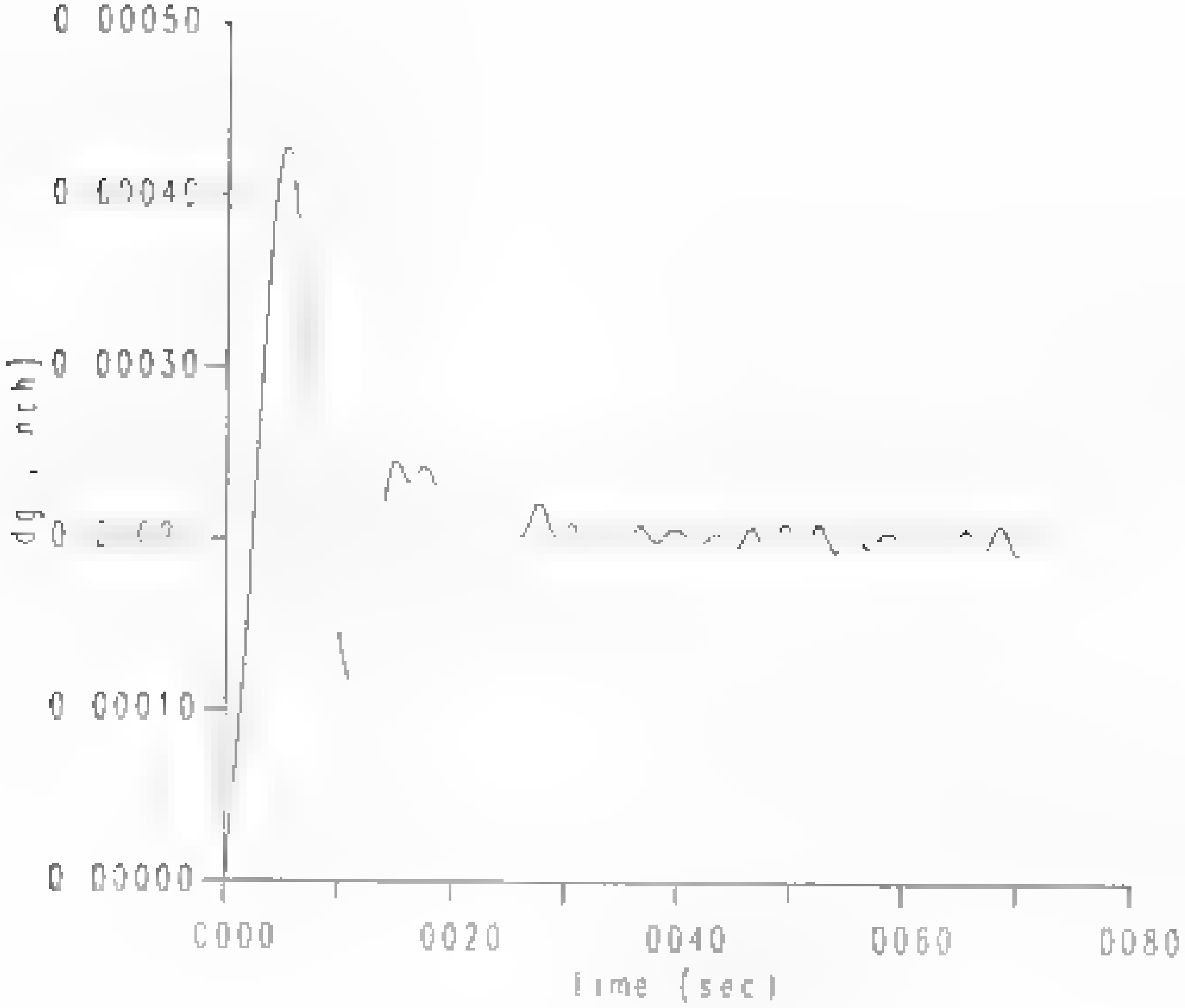


Fig. 3.23 Gear Whirl Subject to Two Unbalances under Subharmonic Resonance with Appropriate Oil Film Bearings (Case 18) $r_r = .00063$, $r_y = .00125$

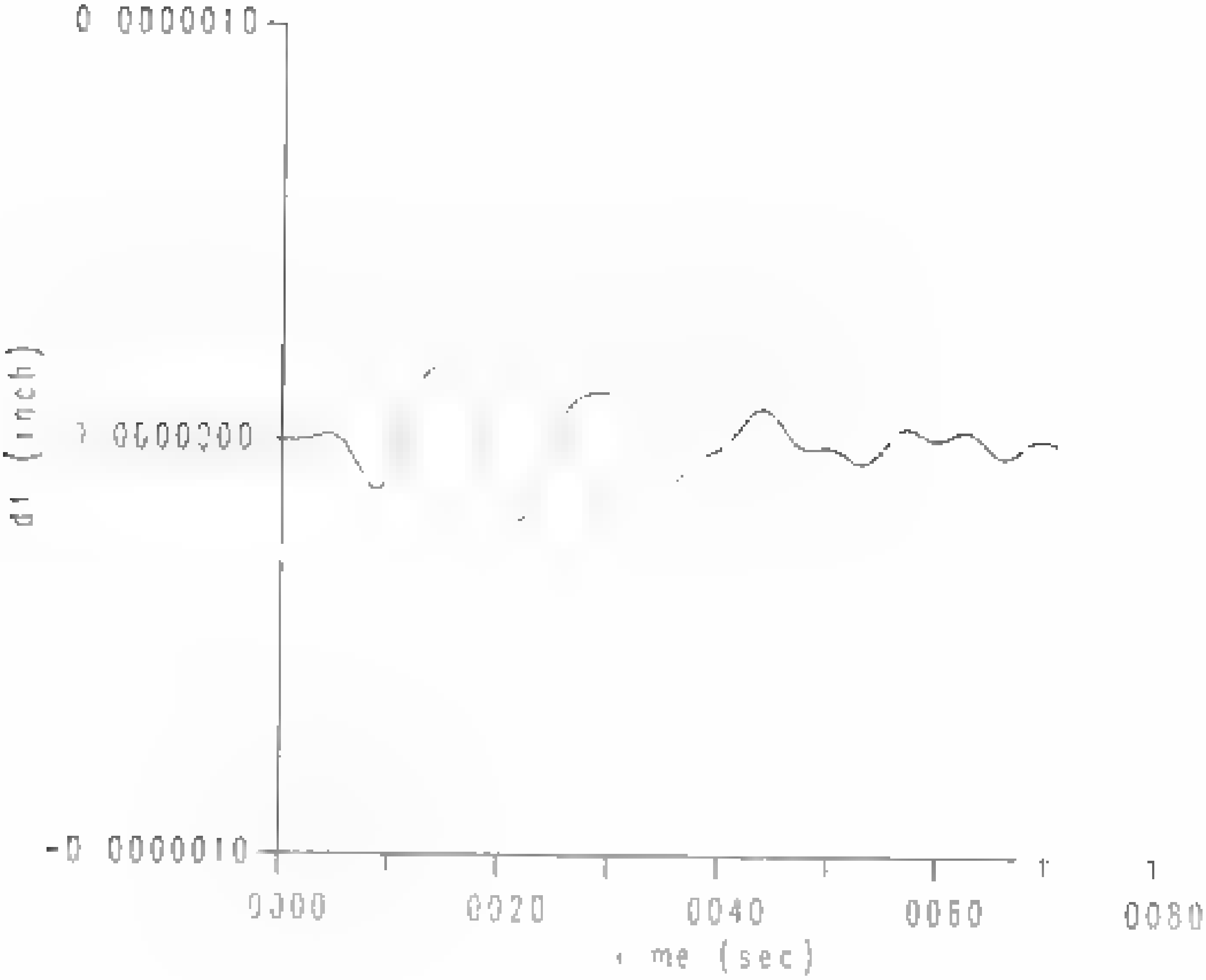


Fig. 3.24 Deflection of Gear Tooth for Case 18
$\tau_r = .00063$, $\tau_y = .00125$

CHAPTER 4

THE USE OF ONE TYPE OF INSTABILITY TO SUPPRESS ANOTHER TYPE

The two mass pinion-gear system (Fig. 2.1) is considered here. The shafts are originally supported on ball bearings. In this chapter, an unbalance is used to suppress the oscillation due to a torsional excitation.

It is conceptually possible to suppress a torsional oscillation by a lateral oscillation since both the lateral and the torsional vibrations of a geared system interact at the points of tooth contact during the mesh action.

The examples discussed in this chapter are considered to illustrate the feasibility of utilizing the concept in the design of high speed gear systems where instabilities are expected to occur.

It should be noted that, in the previous chapter, the additional input is of the same type as the original disturbance. This implies that the effect of the oil film bearings on the individual responses due to the two inputs is basically the same. Therefore, oil film bearings can be added if necessary after investigating the effect of the additional input. However, when a different type of input is imposed to the system, the effect of the oil film bearings on the two responses may not be the same. The system should therefore be analyzed prior to the introduction of the additional input to evaluate the effect of the oil film bearings on the response.

Stable System

In this case, if the amplitude of the response is allowable, then no additional treatment is necessary. In this section, however, the possibility of suppressing the response amplitude due to an existing torsional excitation on one of the gears by introducing an unbalance on the other gear is investigated.

Case 19: System Response

The stable response of the system (Fig. 4.1) due to an oscillatory torque input is evaluated for the same system as in the previous chapter (Fig. 2.1).

The computation is undertaken for the following conditions.

$T_1 = W_t$ (1000) lb-in

ω_{ex} = 3750 rad/sec

ω_r = 3750 rad/sec

ω_{no} = 5012 rad/sec

Case 20: Effect of the Oil Film Bearings

As discussed before, the bearing parameters (length, oil viscosity, and clearance) are selected to give the amount of damping necessary to reduce the amplitude of oscillation of the pinion-gear system due to the torque input. The following parameter values are selected.

$$c = .005"$$

$$\mu = 1.36/(10)^8 \text{ reyns}$$

The tooth spring deflection in this case is shown in Fig. 4.2. It can be seen that damping from oil film has eliminated the beat in the response as well as reduced the peak amplitude.

Case 21: The Selection of a "Tuned" Whirl and its Effect on the System Response

It is necessary in this case that the response due to the imposed unbalance have same frequency as the response due to the torsional excitation.

In order for the response due to the unbalance to exhibit the same frequency as the torsional response, the rotating frequency of the shaft is selected to be the same as the torsional excitation frequency. Also, the amplitudes of the response due to the two different excitations should be of similar magnitude. Considering these factors, the following parameters of the lateral vibrations are selected in an iterative way to obtain a tuned whirl.

$$e_1 = .00009/M_1 \text{ inch}$$

$$\omega_r = 3750 \text{ rad/sec}$$

The tooth spring deflection for the system supported on the same oil film bearings due to the imposed unbalance excitation is shown in Fig. 4.3.

This unbalance can be introduced in the system with 180 degrees of phase shift which is found to provide the most reduction in the amplitude of oscillation due to the torsional input. As shown in Fig. 4.4, the amplitude of the tooth spring deflection is suppressed by the tuned whirl, and the gear whirl exhibits decay instead of build-up (Fig. 4.5).

System with Torsional Resonance

Resonance in this example is obtained by selecting the excitation frequency to be the same as the nominal natural frequency of the pinion-gear system i.e.,

$$\omega_{ex} = \omega_{no} = k_o \frac{(J_1 + J_3)}{J_1 \; J_3} \qquad 4.1$$

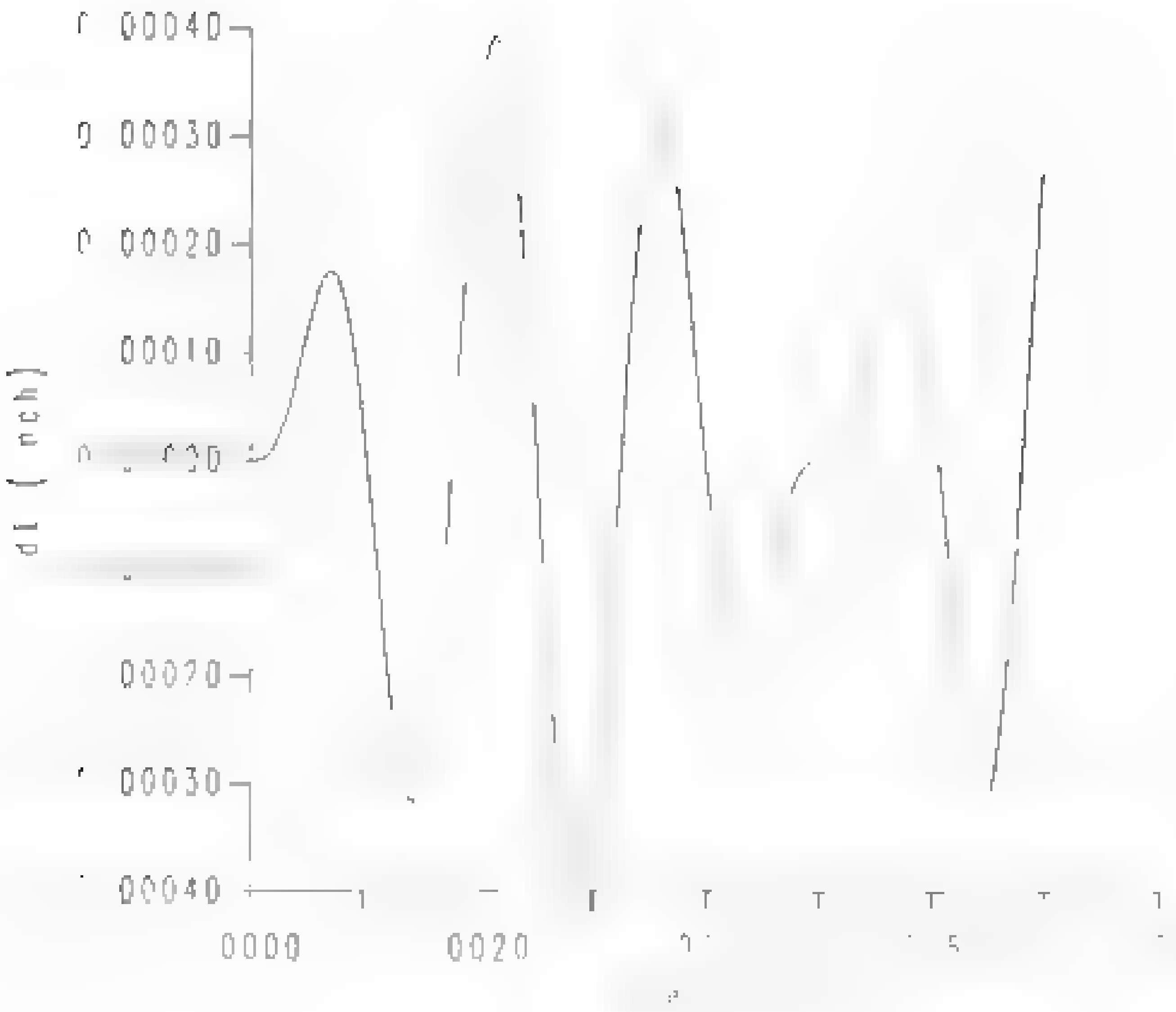


Fig. 4.1 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Rotor 1 (Case 19) τ_{ex} = .0017, τ_{no} = .00125

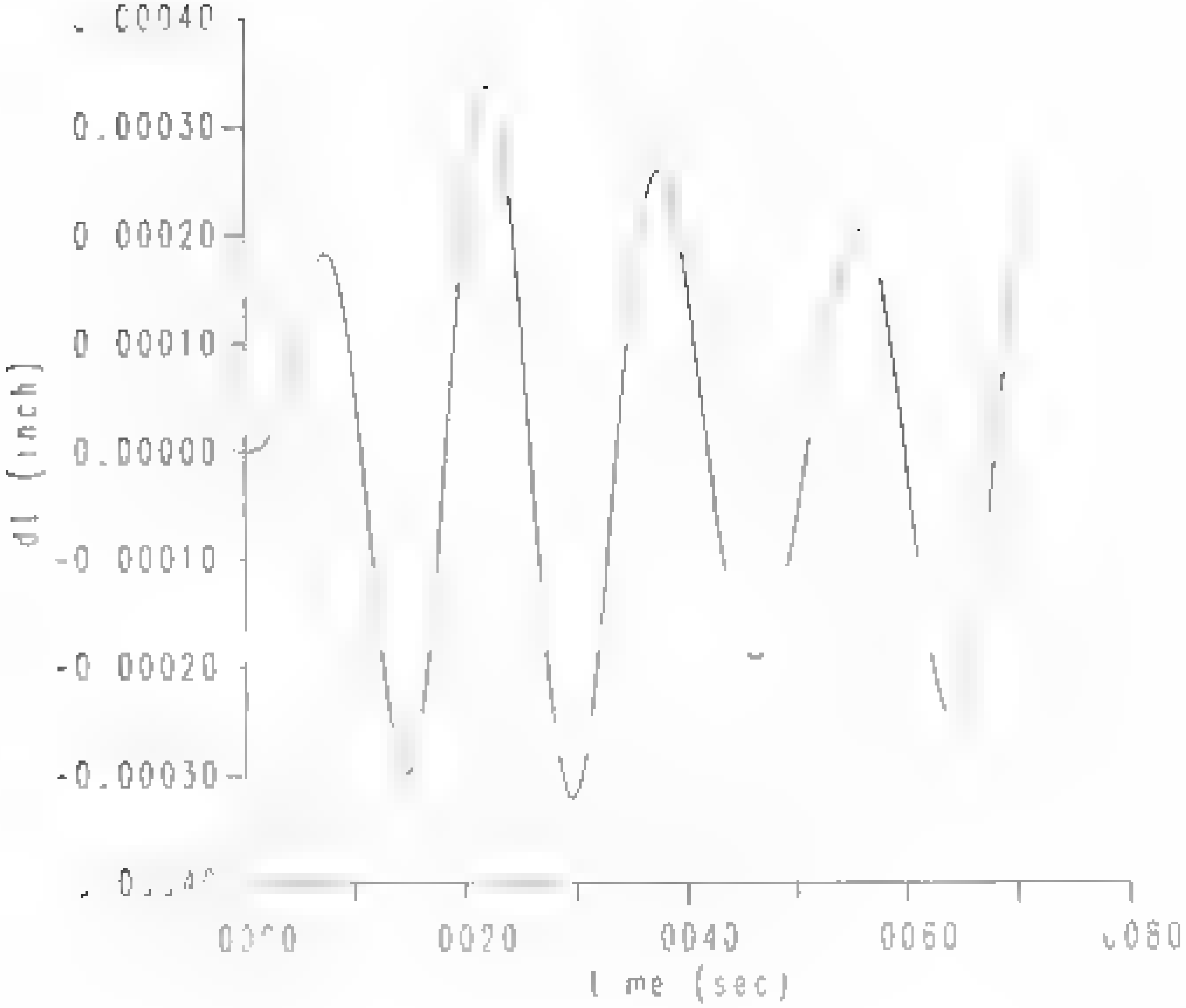


Fig. 4.2 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Rotor 1 with the Effect of Oil Film Bearings (Case 20) $\tau_{ex} = .0017$, $\tau_{no} = .00125$

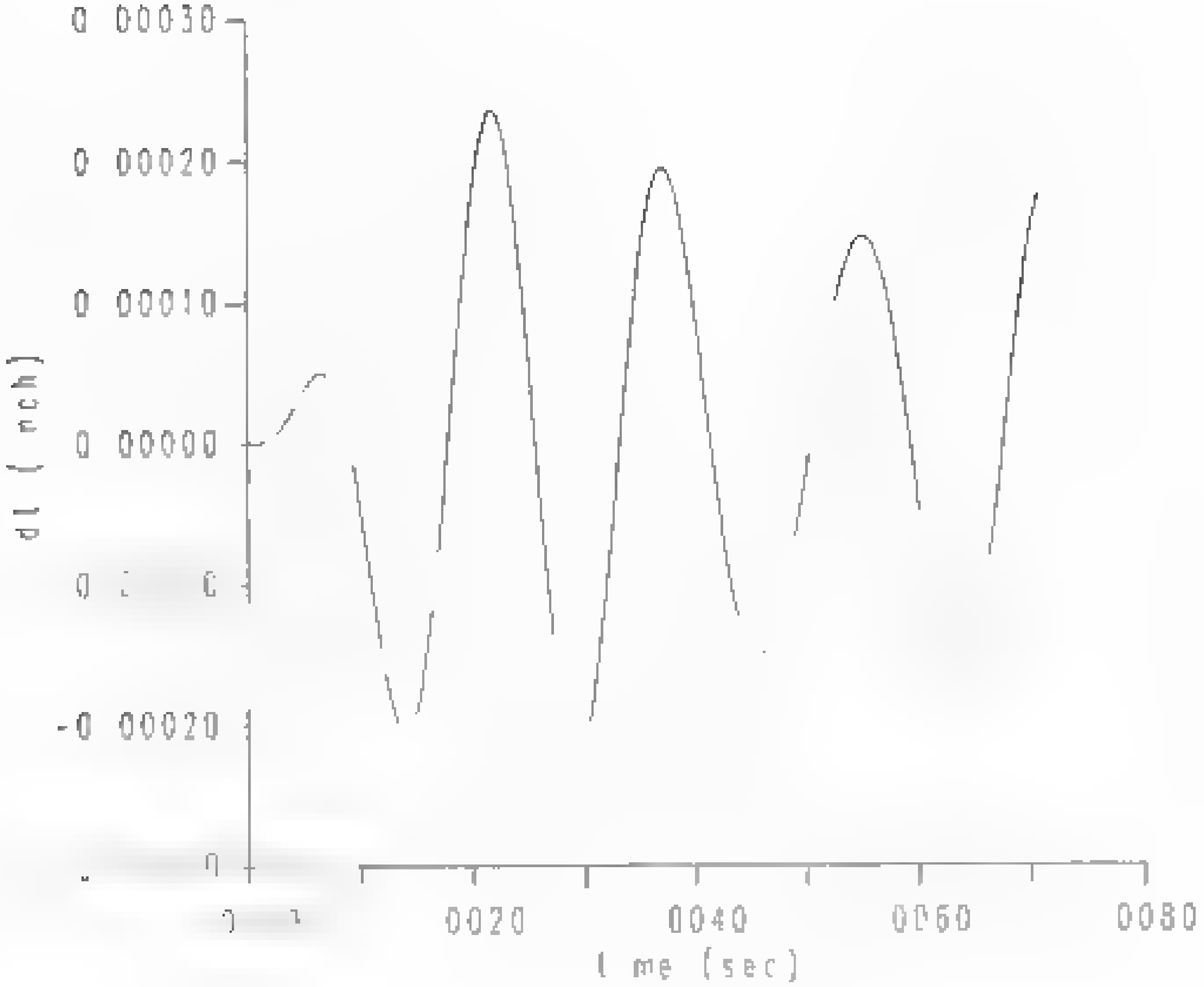


Fig. 4.3 Deflection of Gear Tooth at the Mesh due to an Unbalance Excitation at Rotor 3 with the Effect of Oil Film Bearings (Case 21) $\zeta_x = .0017$, $\zeta_y = .00125$

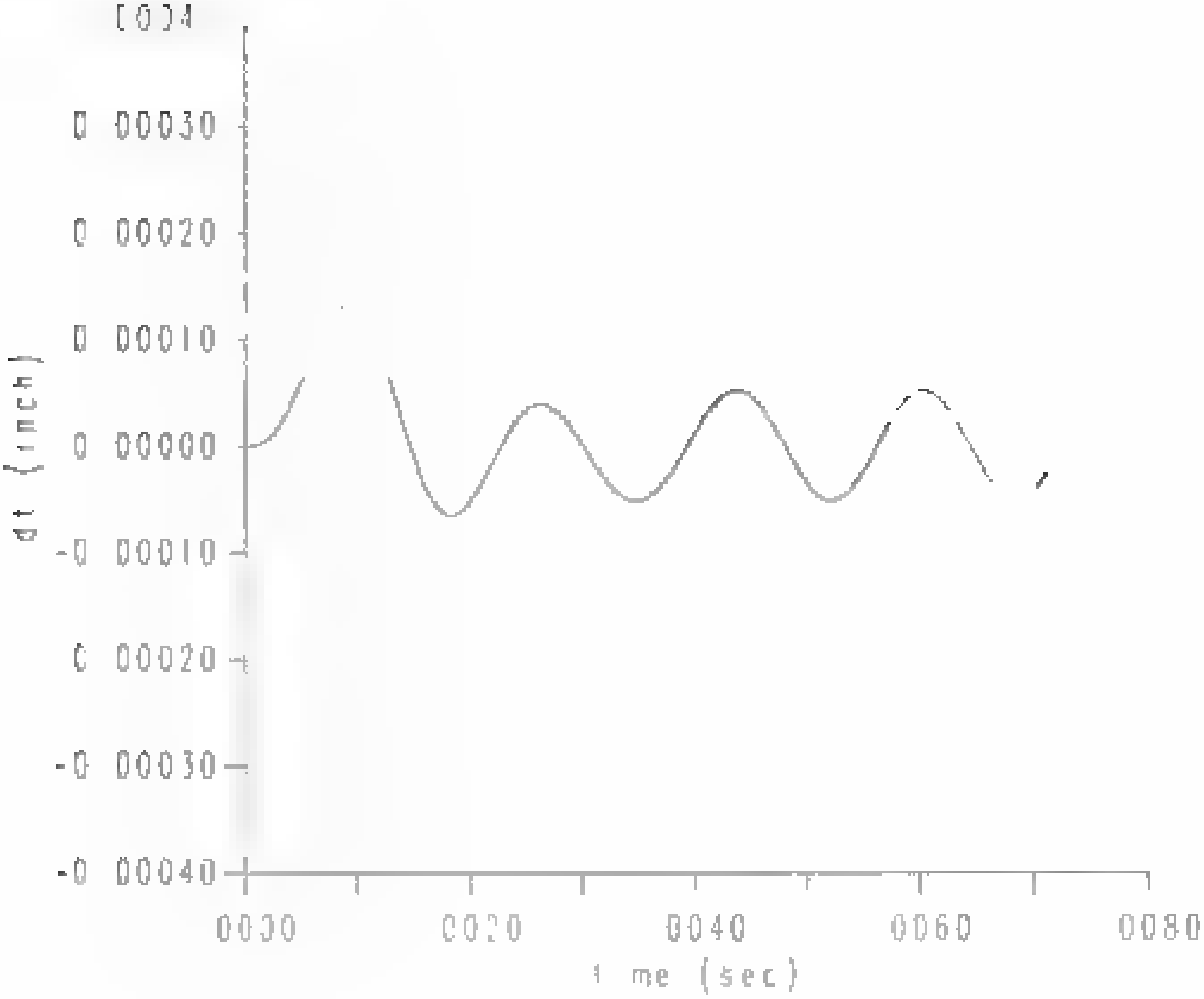


Fig. 4.4 Deflection of Gear Tooth at the Mesh due to the Two Excitations with Proper Phase Shift with the Effect of Oil Film Bearings (Case 21) $\tau_{ex} = .0017$, $\tau_{no} = .00125$, $\tau_{y} = .00125$

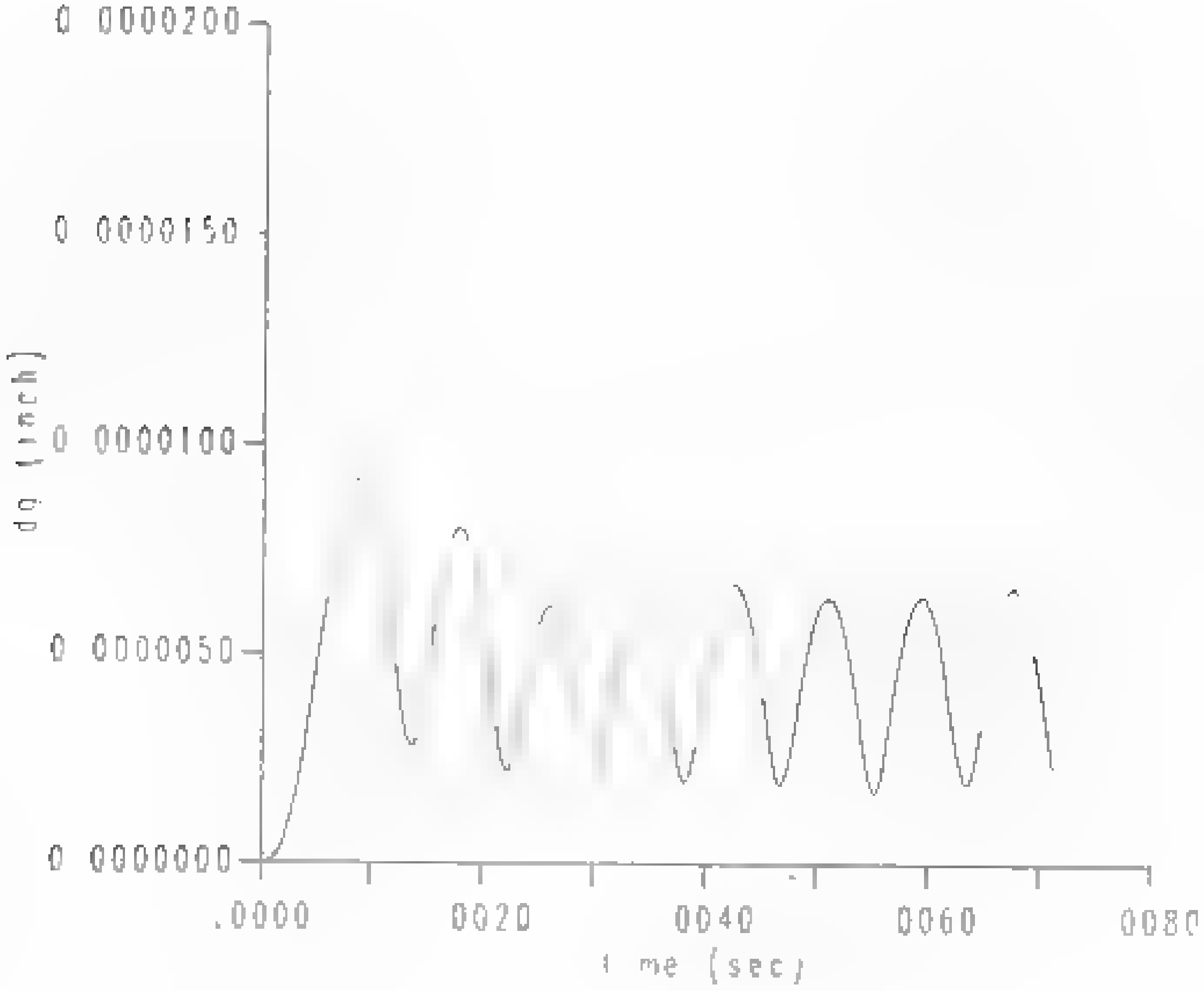


Fig. 4.5 Gear Whirl for Case 21
$\tau_{ex} = .0017$, $\tau_{no} = .00125$,
$\tau_{y} = .00125$

Case 22: Resonant Response

The resonant response of the system due to a torque excitation builds up linearly with time as would be expected.

The response shown in Fig. 4.6 is obtained for the following conditions.

$$T_1 = W_t \ (1000) \text{ lb-in}$$
$$\omega_{ex} = \omega_{no} = 5012 \text{ rad/sec}$$
$$\omega_r = 5000 \text{ rad/sec}$$

It can be seen from the figure that the response build-up is at a frequency which is the same as the nominal natural frequency of the pinion-gear system.

Case 23: The Effect of the Oil Film Bearings

The effect of the damping in the oil film bearing is examined first. Bearing length, oil viscosity, and clearance of the bearing are in the same way as in the previous cases to provide positive damping. For this case, the following parameter values are selected.

$$c = .006"$$
$$\mu = 1.36/(10)^8 \text{ reyns}$$

As shown in Fig. 4.7, the build-up is suppressed by the positive damping provided by the oil film.

Case 24: The Selection of the "Tuned" Whirl and its Effect on System Response

The resonant response of the same system due to the unbalance can be induced to have the same form as that due to the fluctuating torque input when the torque exciting frequency is the same as the rotational frequency. The following inputs are used in the analysis.

$e_3 = .000012/M_1$ inch

$\omega_r = 5000$ rad/sec

$\omega_y = 5029$ rad/sec

$\omega_{ex} = \omega_{no} = 5012$ rad/sec

The calculated tooth spring deflection for the system supported on the same oil film bearing with tuned whirl is shown in Fig. 4.8.

The oscillation due to an unbalance can be introduced to the system with a 61.9 degrees of phase lag which is determined iteratively to suppress the resonance due to the torque input. As shown in Fig. 4.9, the amplitude of the tooth spring deflection is significantly reduced by the additional tuned input. Also, the gear whirl caused by oil film bearing and the unbalance excitation attains a stable behavior with a limit cycle (Fig. 4.10).

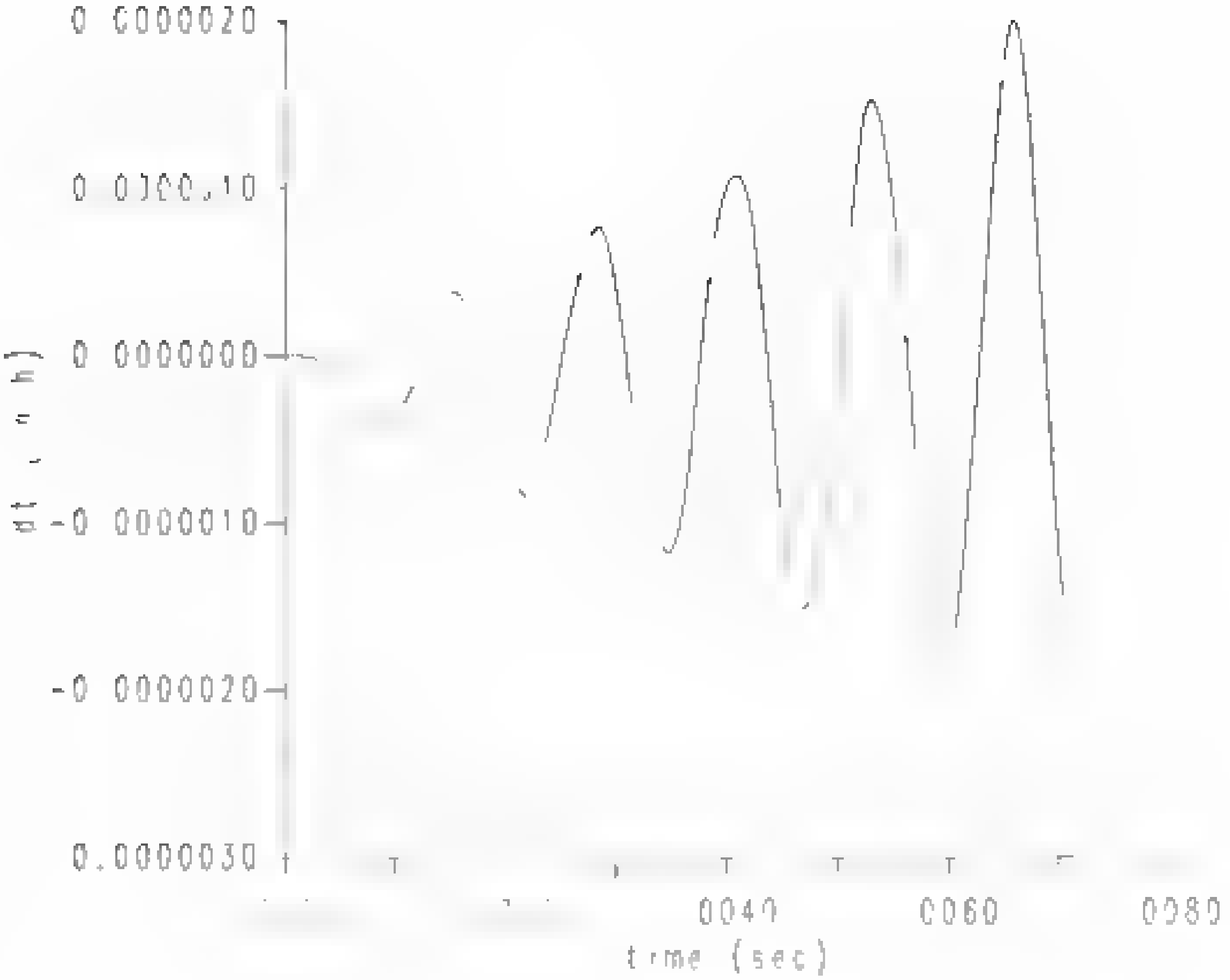


Fig. 4.6 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Resonant Frequency at Rotor 1 (Case 22) $\tau_{ex} = .00125$, $\tau_{no} = .00125$

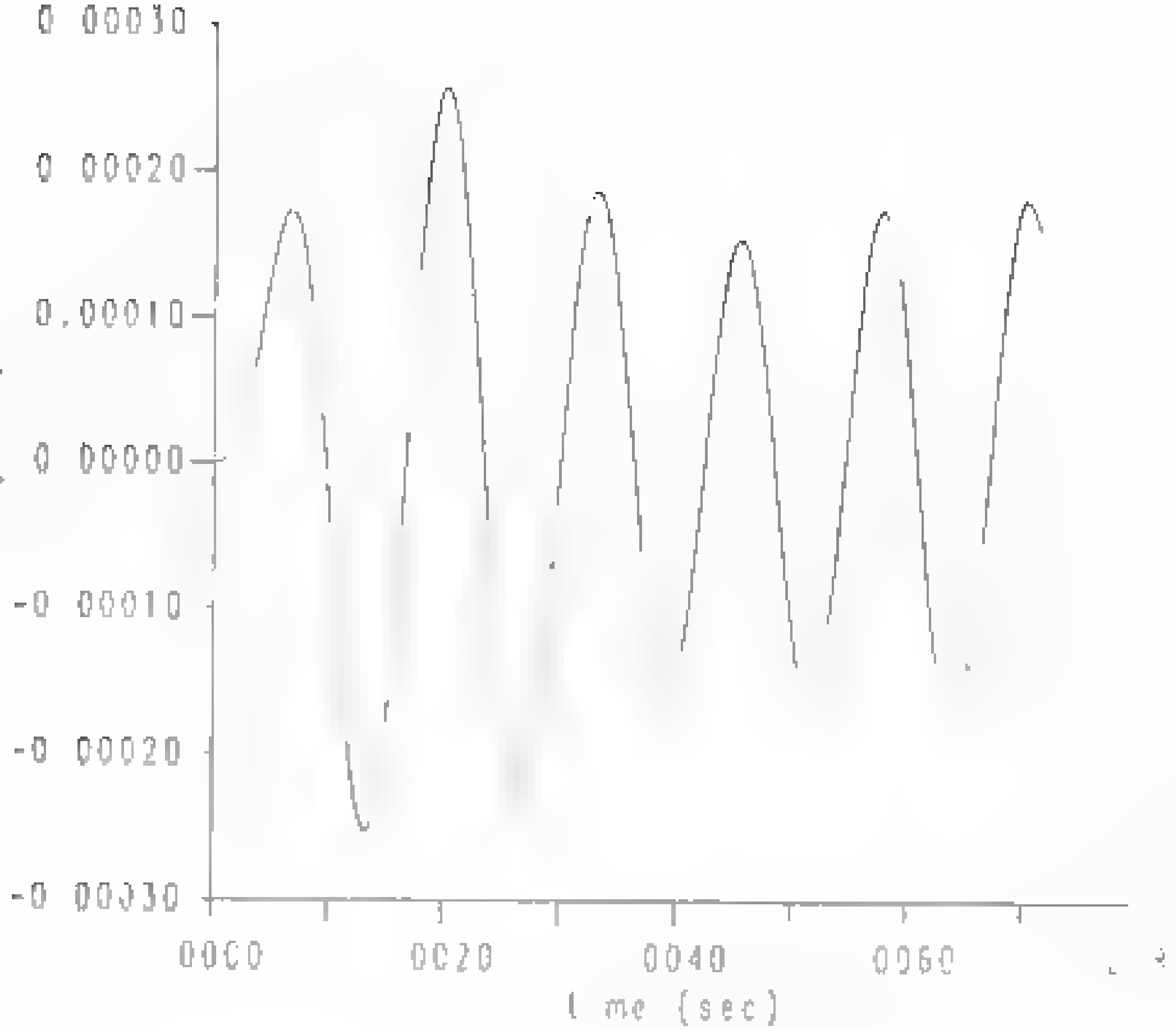


Fig. 4.7 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Resonant Frequency at Rotor 1 with the Effect of Oil Film Bearings (Case 23) $f_{ex} = .00125$, $f_{no} = .00125$

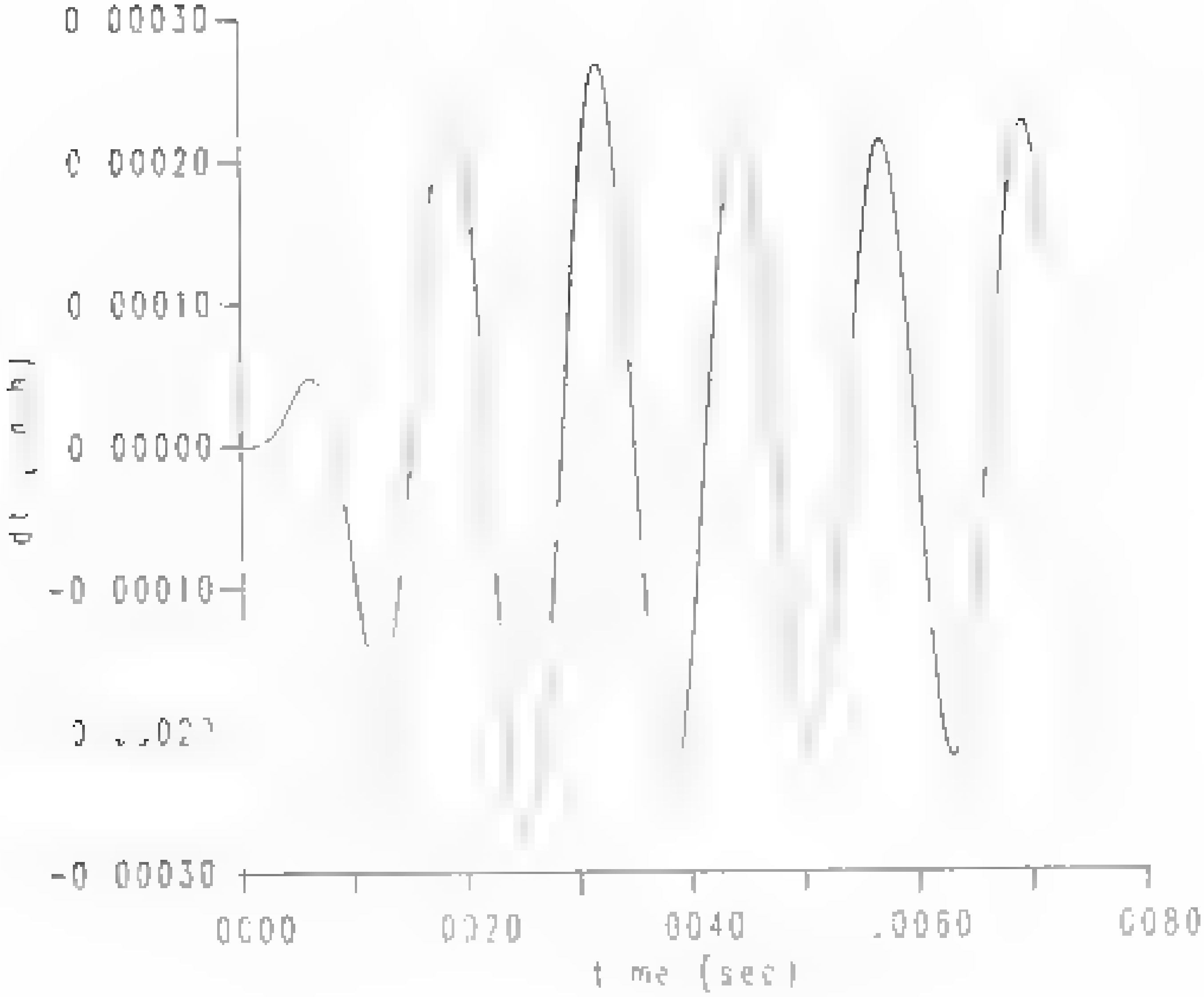


Fig. 4.8 Deflection of Gear Tooth at the Mesh due to an Unbalance Excitation at Resonant Frequency at Rotor 3 with the Effect of Oil Film Bearings (Case 24)
τ_r = .00125, τ_y = .00125

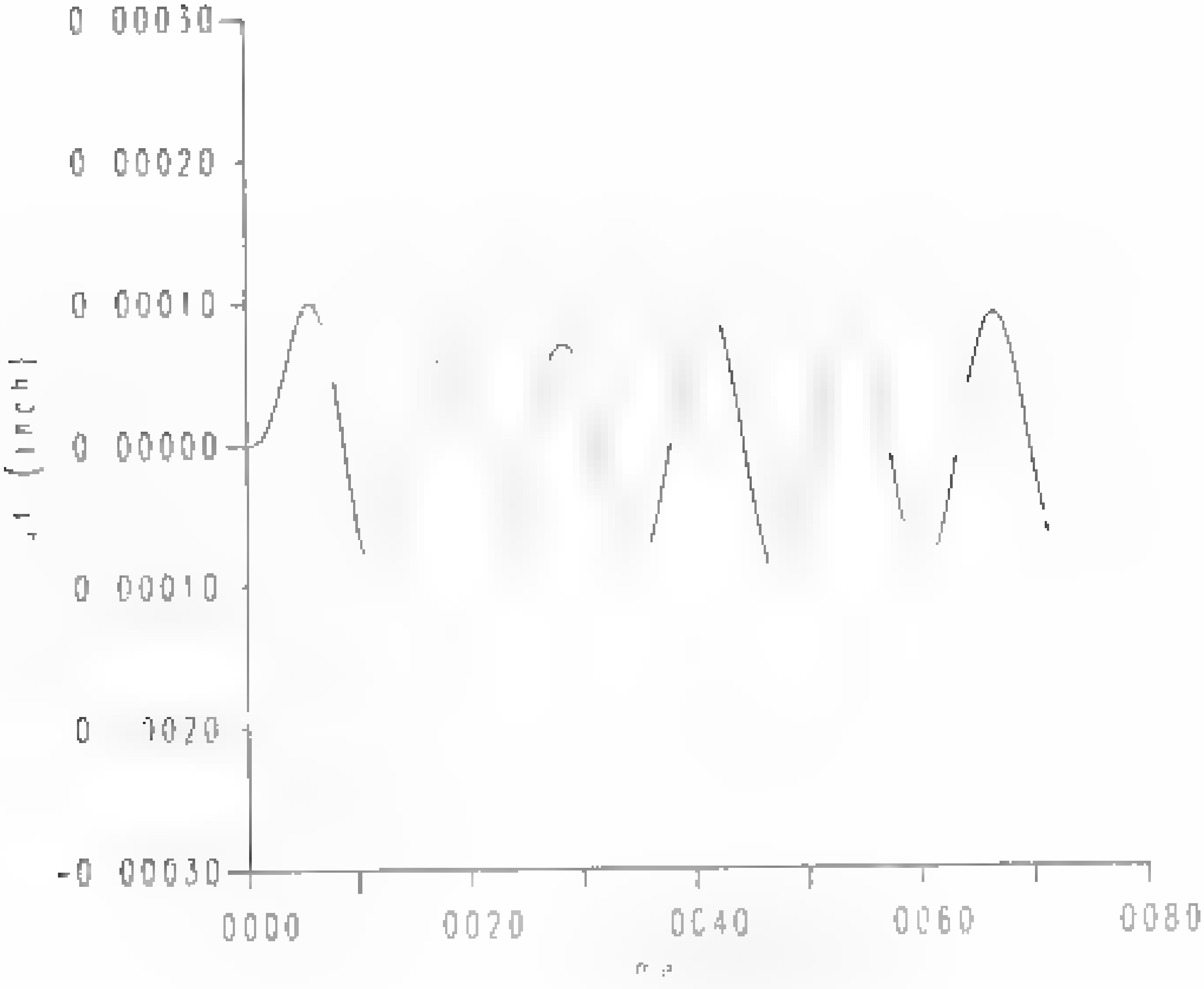


Fig. 4.9 Deflection of Gear Tooth at the Mesh due to the Two Excitations at Resonant Frequency with Proper Phase Shift with the Effect of Oil Film Bearings (Case 24)
$\zeta_{ex} = .00125$, $\zeta_{no} = .00125$, $\zeta_{y} = .00125$

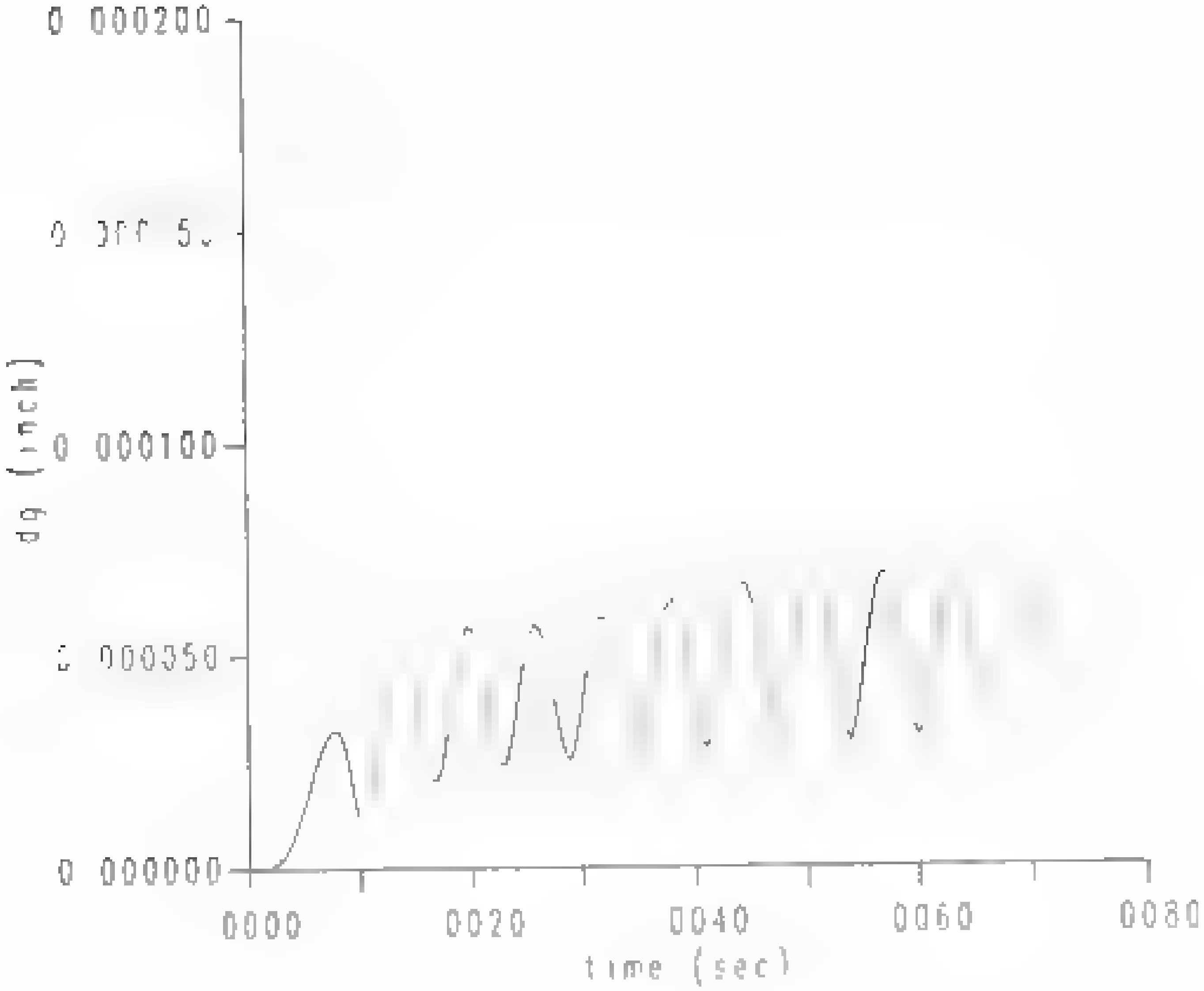


Fig. 4.10 Gear Whirl for Case 24
$f_{ex} = .00125$, $f_{no} = .00125$,
$f_y = .00125$

System with Torsional Instability

In this section, the same torsional instability which was discussed in section 3.1 is considered here to investigate the effect of introducing an unbalance on the rotor in order to suppress it. Again, the unstable situation, characterized by the exponential build-up, is expected when

$$\frac{\omega_{ex}}{\omega_{no}} = 2 \frac{\omega_{ex}}{\omega_m} \qquad 3.4$$

Case 25: Unstable Response

The same unstable response as in section 3.1. is obtained when the condition which satisfies Eq. 3.2 exists between excitation frequency of the torque, the nominal torsional natural frequency of the pinion-gear system, and the mesh frequency. The following input parameters are used.

$T_1 = W_t(1000)$ lb-in

$\omega_{ex} = 2500$ rad/sec.

$\omega_{no} = 5012$ rad/sec

$\omega_m = 10000$ rad/sec

As shown in Fig 4.11, the instability is characterized by the exponential build-up pattern.

Build-up frequency is same as natural frequency even though the excitation frequency is one half of that frequency, which is expected in the considered case of superharmonic resonance.

Case 26: Effect of the Oil Film Bearings

As in previous cases, the effect of damping in the oil film bearings on the response is investigated first. The bearing parameters are selected based on the same scheme as discussed in sections 3.1 and 3.2. For this case, the following parameter values are used.

$c = .004"$

$\mu = 1.36/(10)^8$ reyns

As shown in Fig. 4.12, the response of the system reaches a limit cycle due to the influence of oil film.

Case 27: The Selection of the "Tuned" Whirl and its Effect on the System Response

The response of the system supported on the same oil film bearing due to the unbalance excitation (Fig. 4.13) is expected to have the same vibration frequency as the response due to torque input for the following conditions.

$e_3 = .00006/M_1$ inch

$\omega_r = 2500$ rad/sec

$\omega_y = 5029$ rad/sec

The additional oscillation created by the unbalance excitation is introduced in the system with a 32.8 degrees of phase lag which is determined iteratively. The resultant tooth spring deflection is shown in Fig. 4.14, and Fig. 4.15 shows the gear whirl caused by the unbalance.

In sections 3.1, and 3.2, the additional inputs used to improve the system stability are of the same type as those causing the problems. However, in this chapter, it is shown that different type of excitation can be used to suppress the adverse effects of other disturbances. In other words, it is possible to introduce an appropriate amount of unbalance with the proper phase shift can be introduced in the system to suppress large oscillatory amplitudes arising from resonances or instabilities generated by torque excitations.

The investigated cases show that the potentially destructive effect from a torque excitation can be avoided by "tuned" whirl and the proper design of oil film bearing without causing a large amount of lateral gear movement. All the reported results have been obtained without considering the effect of the inherent damping in the mechanical system itself.

It can therefore be stated that it is possible in many situations to rely on the appropriate coupling of torsional and lateral instabilities to neutralize the adverse effects which can result from either of them if they would occur separately.

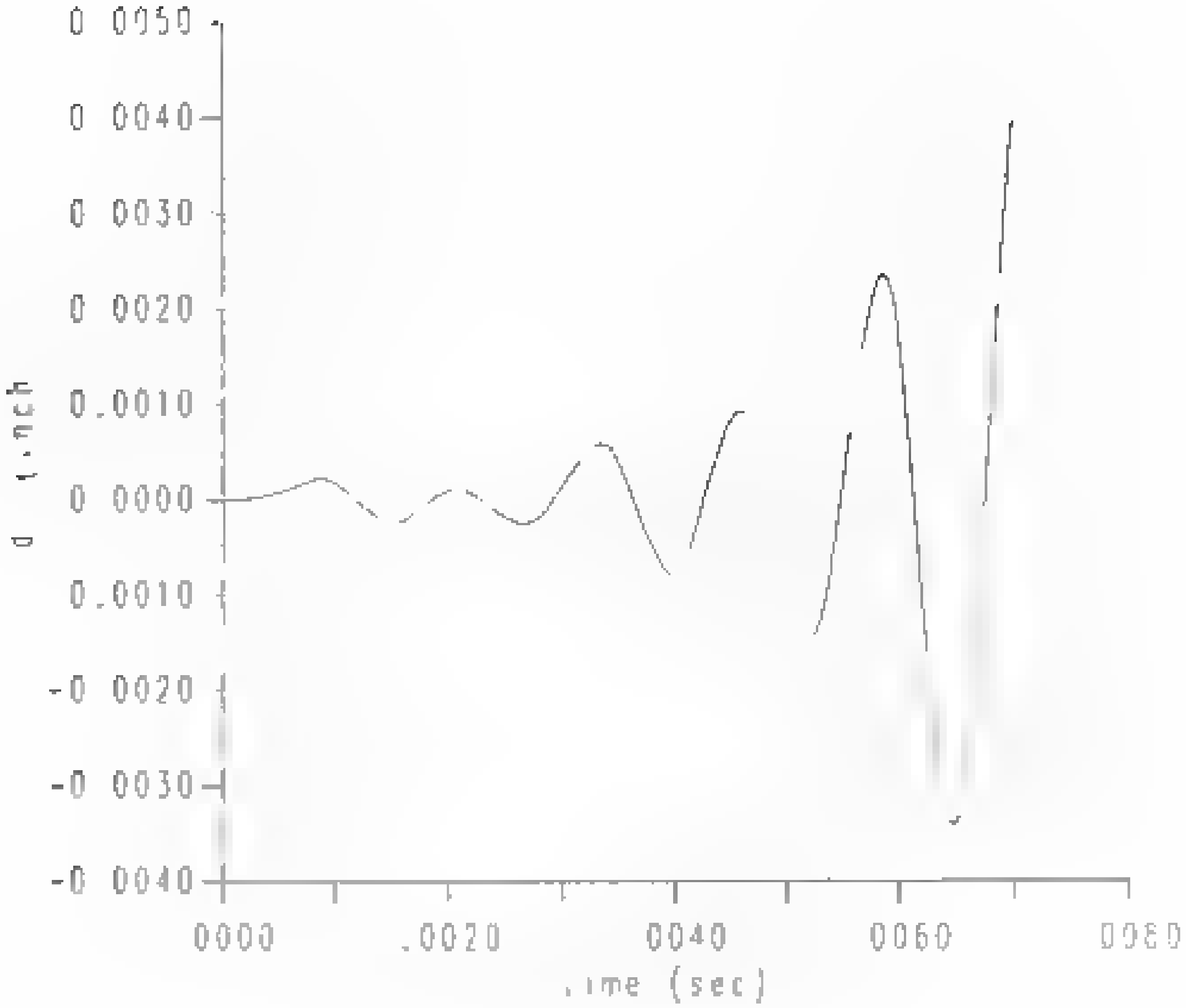


Fig. 4.11 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Rotor 1 for Unstable Condition (Case 25) τ_{ex} = .0025, τ_{no} = .00125

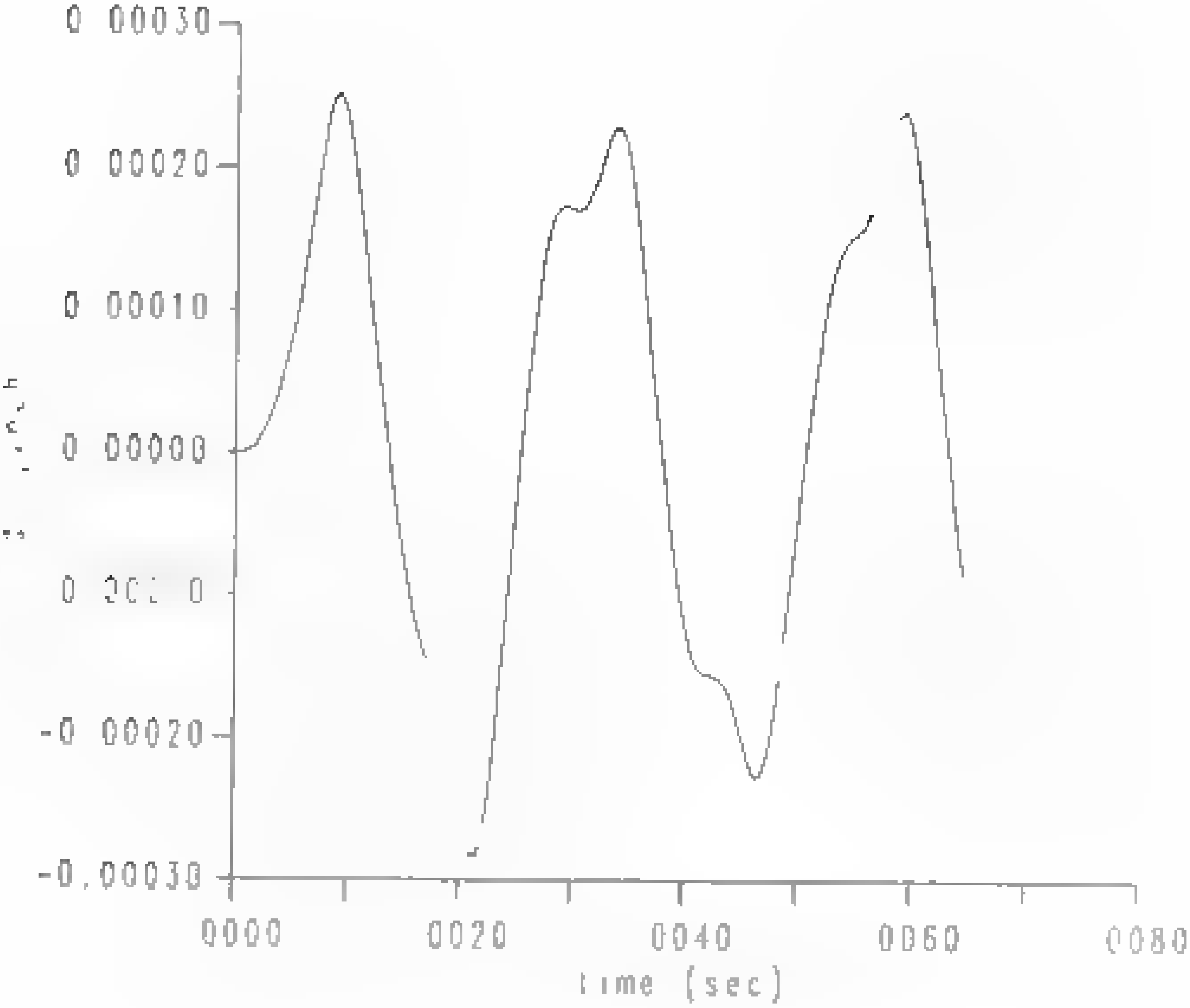


Fig. 4.12 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Superharmonic Resonant Frequency at Rotor 1 with the Effect of Oil Film Bearings (Case 26)
τ_{ex} = .0025, τ_{no} = .00125

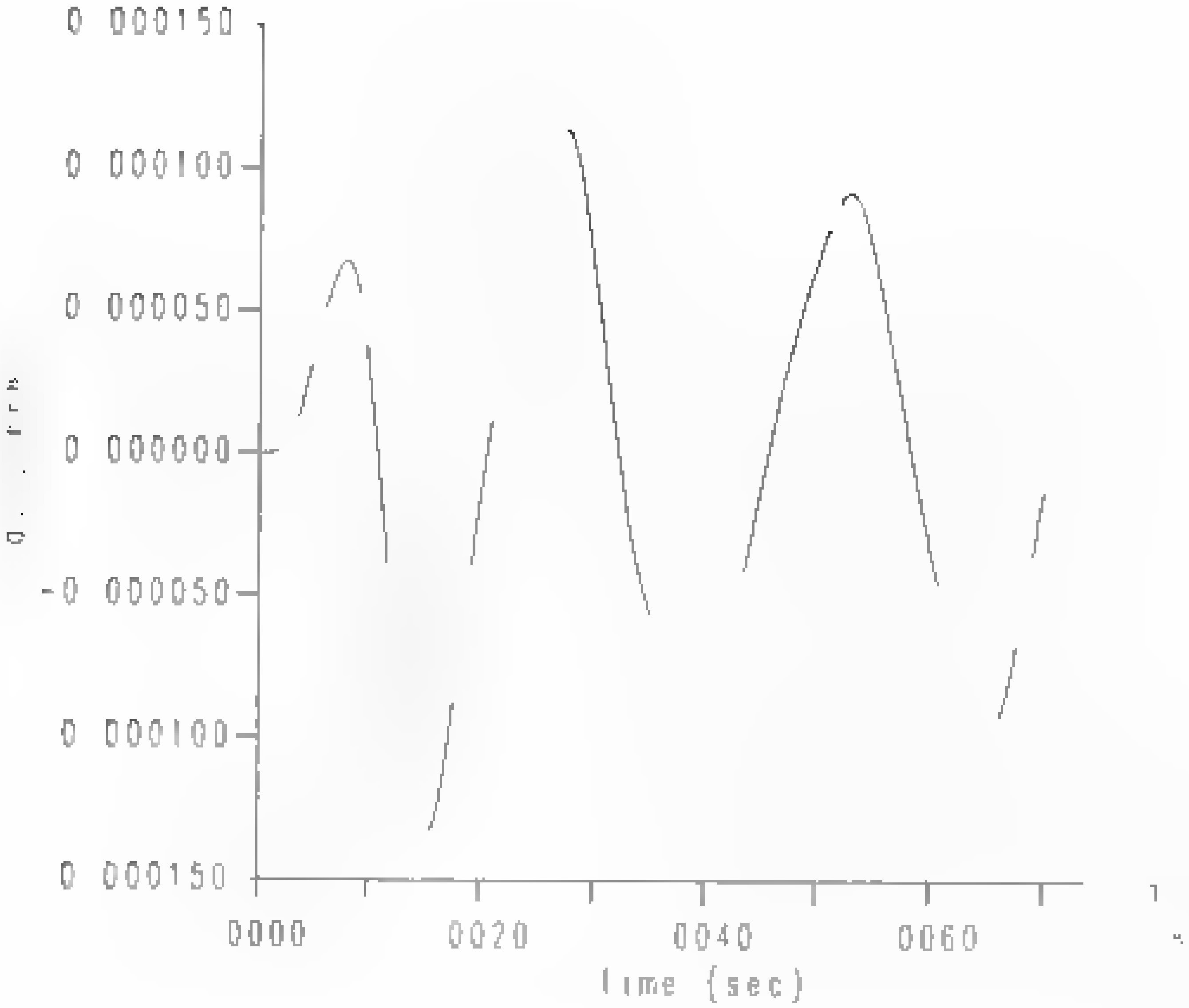


Fig. 4.13 Deflection of Gear Tooth at the Mesh due to an Unbalance Excitation at Superharmonic Resonant Frequency at Rotor 3 with the Effect of Oil Film Bearings (Case 27)
$\epsilon_r = .0025$, $\epsilon_y = .00125$

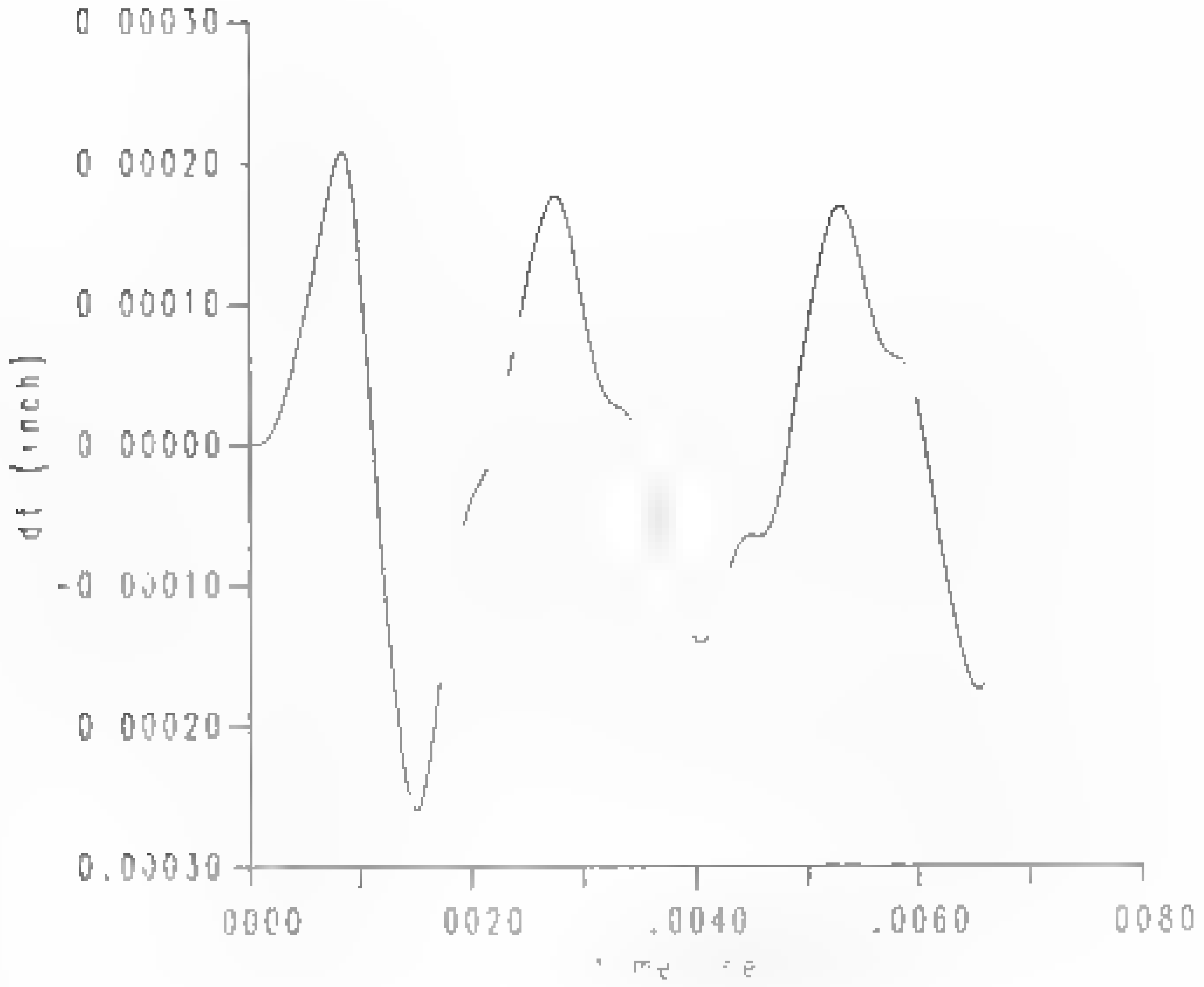


Fig. 4.14 Deflection of Gear Tooth at the Mesh due to the Two Excitations at Superharmonic Resonant Frequencies with Proper Phase Shift with the Effect of Oil Film Bearings (Case 27) τ_{ex} = .0025, τ_{no} = .00125, τ_{y} = .00125

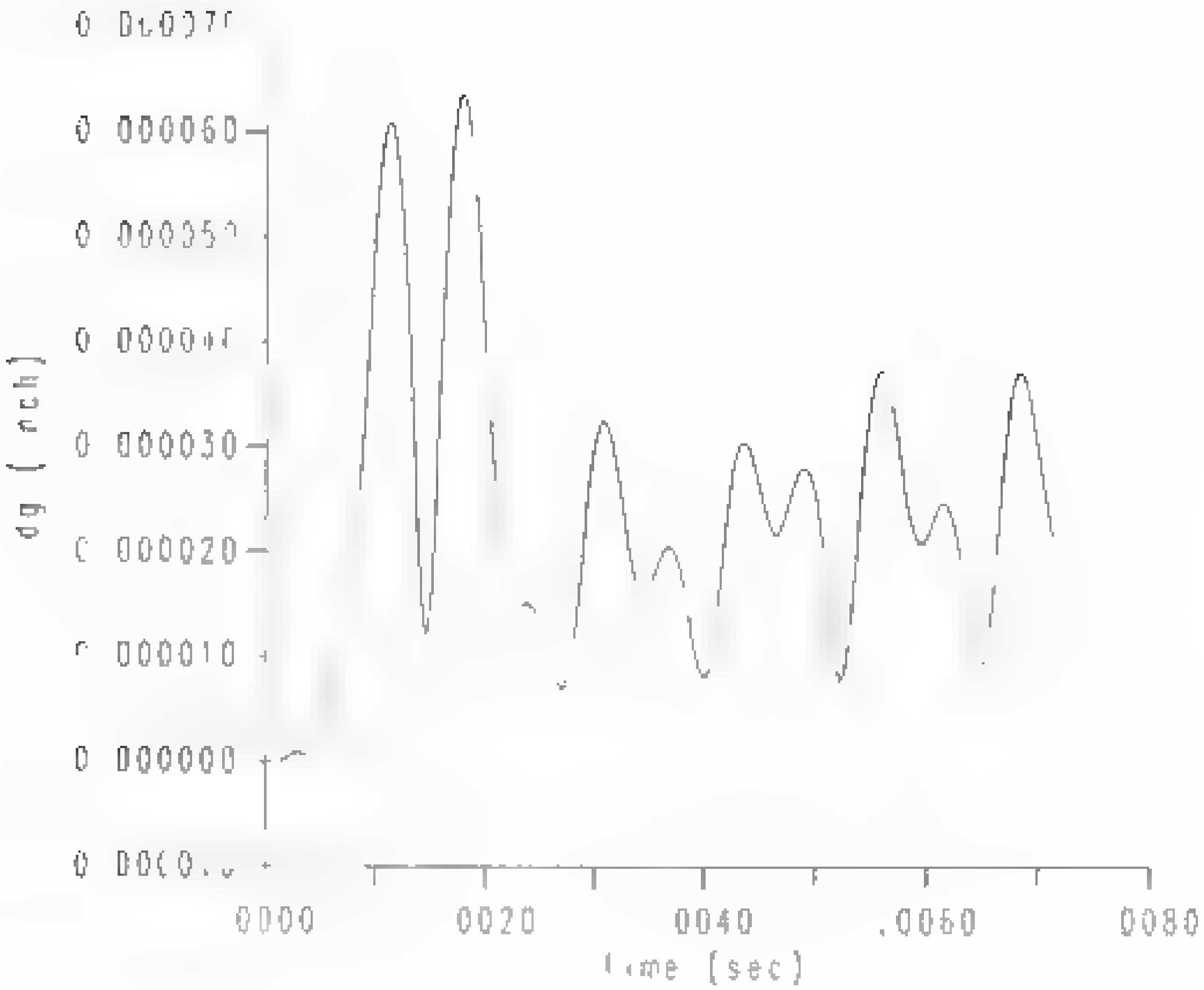


Fig. 4.15 Gear Whirl for Case 27
$\tau_{ex} = .0025$, $\tau_{no} = .00125$,
$\tau_y = .00125$

CHAPTER 5

APPLICATION TO GEARED SYSTEMS WITH TWO OVERHANG MASSES

In this chapter, a whirl excitation due to an unbalanced overhang mass is introduced to a system to neutralize the effect of a torsional instability induced by a torque excitation on the other overhang mass. The objective is to investigate whether the same approach can be applied to complex system with more degrees of freedom than the two rotor system which has been considered so far.

Basically, the system analyzed in this chapter (Fig. 2.2) is the same as the one treated in previous chapters except for the complexity (six more degrees of freedom) generated by introducing two additional rotors. The main distinction is that the effect of the excitation induced on the overhang rotors reaches the contact points on the teeth with some lag which depends on the dynamic parameters of the system.

In order to demonstrate the feasibility of the approach, only the unstable case of exponential build-up of the response will be investigated.

The following are the main system parameters used to investigate the system used in the reported study.

ϕ = 20 degree

N= 4200 rpm

$\omega_1 = 2 \pi N / 60$ rad/sec

$\omega_2 = \omega_1$ (GR) rad/sec

$\omega_m = \omega_2$ (# of teeth of pinion) rad/sec

$D_1 = 2.5"$

$D_2 = 2.5"$

d_b(bearing diameter) = $D_1 + 2 c$ inch

l_b(bearing length) = 2.5"

$L_1 = L_4 = 3"$

$L_2 = L_3 = L_5 = L_6 = 16.2"$

$M_1 = M_3 = .172$ slug

$M_2 = M_4 = .172$ slug

$r_1 = r_2 = 5"$

GR = 1

$W_t = 3.0"$

T = 300 lb-in

$r_3 = r_1 / GR$

$r_4 = r_3$

$k_o = 4865$ lb/in

$E = 30\ (10^6)$ lb/in^2

Poisson's ratio = .292

Case 28: Unstable Response due to a Torque Excitation on an Overhang Mass

An unstable response (Fig. 5.1) with exponential build-up is obtained for the following conditions which causes a superharmonic resonance of the pinion-gear system supported originally by ball bearings.

$$T_2 = 300 \text{ lb-in}$$

$$\omega_{no} = 316 \text{ rad/sec}$$

$$\omega_{ex} = 316 \text{ rad/sec}$$

In this case, the torque is applied on an overhang mass and transmitted to the pinion-gear system through the shaft.

Ca[redacted] 29: Effect of the Oil Film Bearings

Once again, the effect of the oil film bearings on the response is investigated before introducing a tuned whirl. The oil film bearing can be designed in the same way as in the previous cases. For this case the following parameter values are selected.

$$c = .002"$$

$$\mu = 1.36/(10)^8 \text{ reyns}$$

As shown in Fig. 5.2, the tooth spring deflection is stabilized to a limit cycle due to the effect of the positive damping provided by the oil film bearings.

Case [redacted]0: Selection of the "Tuned" Whirl and its Effect on the System Response

As in the previous cases, a whirl response with the same frequency (Fig. 5.3) is necessary to suppress the instability due to the torque. In this chapter the tuned response is obtained from the proper amount of eccentricity on the other overhang mass with a 150 degrees of phase lag which is obtained in an iterative way.

$e_4 = .08/M_4$ inch

$\omega_r = 440$ rad/sec

The effect of the unbalance introduced in an overhang mass as well as the effect of the torque on the other overhang mass is transmitted with phase lags to the tooth mesh where the lateral and the torsional oscillations interact.

As in the previous case of the two rotor geared system, the parameters are modified to match the frequencies of the system response to the torque input and the unbalance excitations.

Using the two excitations along with the proper amount of the phase shift, the system shows very low response amplitude (Figs. 5.4 and 5.5)

It can therefore be seen that the additional tuned whirl induced by the unbalanced overhang mass and the proper design of the oil film bearings can eliminate the potentially destructive instability due to the torque excitation applied on the other overhang mass.

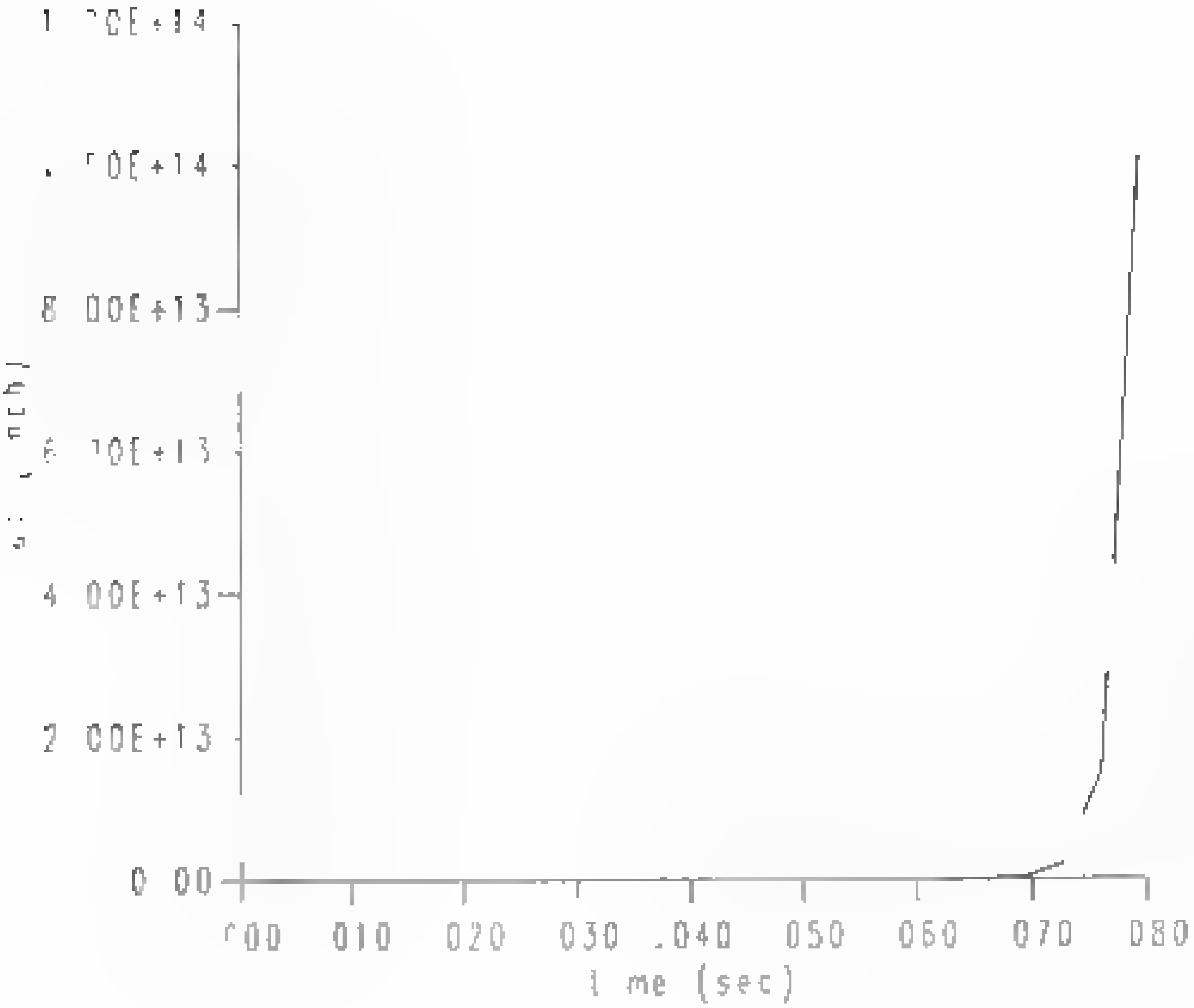


Fig. 5.1 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Rotor 2 for Unstable Condition (Case 28) $f_{ex} = .02$, $f_{no} = .02$

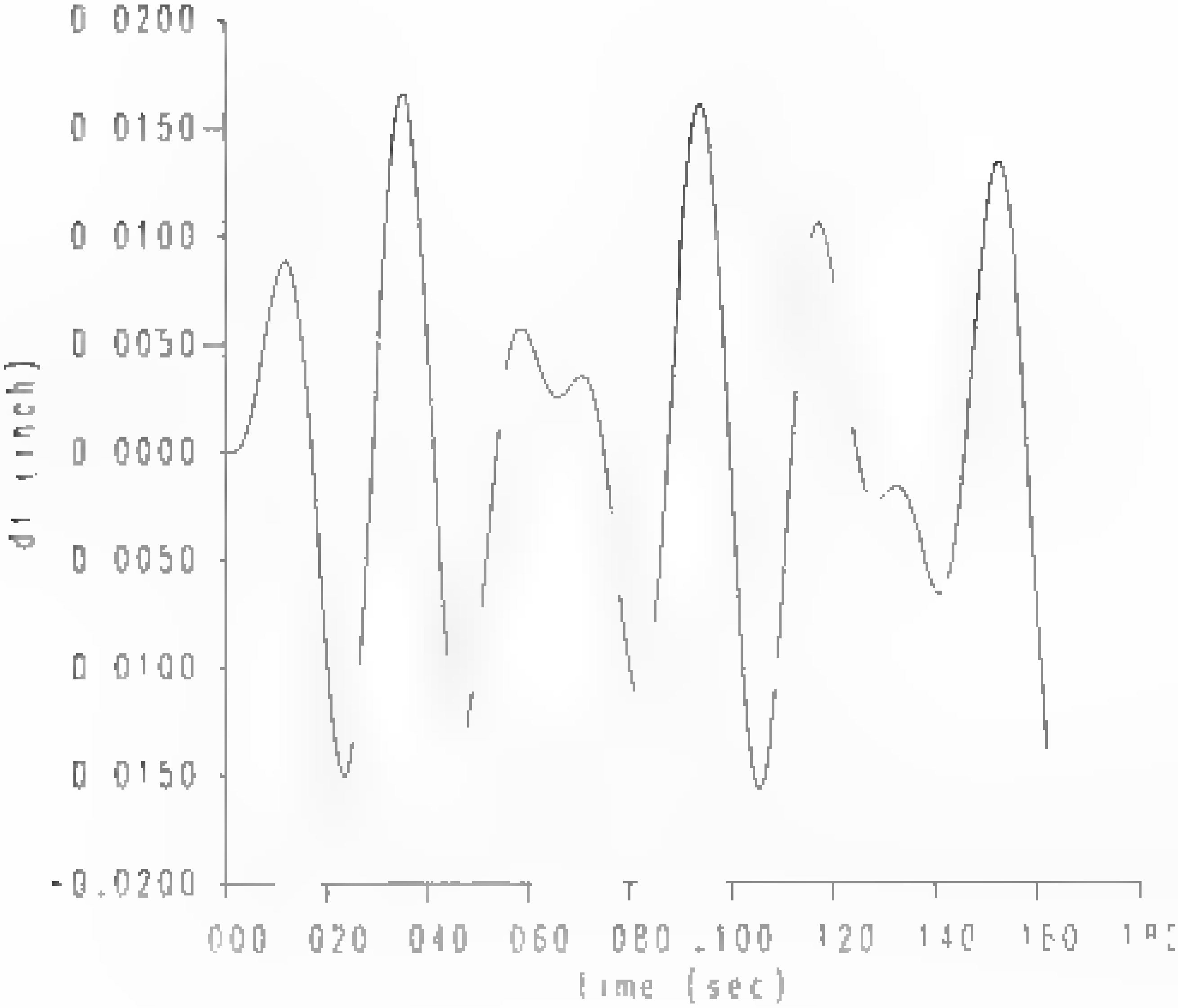


Fig. 5.2 Deflection of Gear Tooth at the Mesh due to a Torsional Excitation at Rotor 2 with the Effect of Oil Film Bearings (Case 29) τ_{ex} = .02, τ_{no} = .02

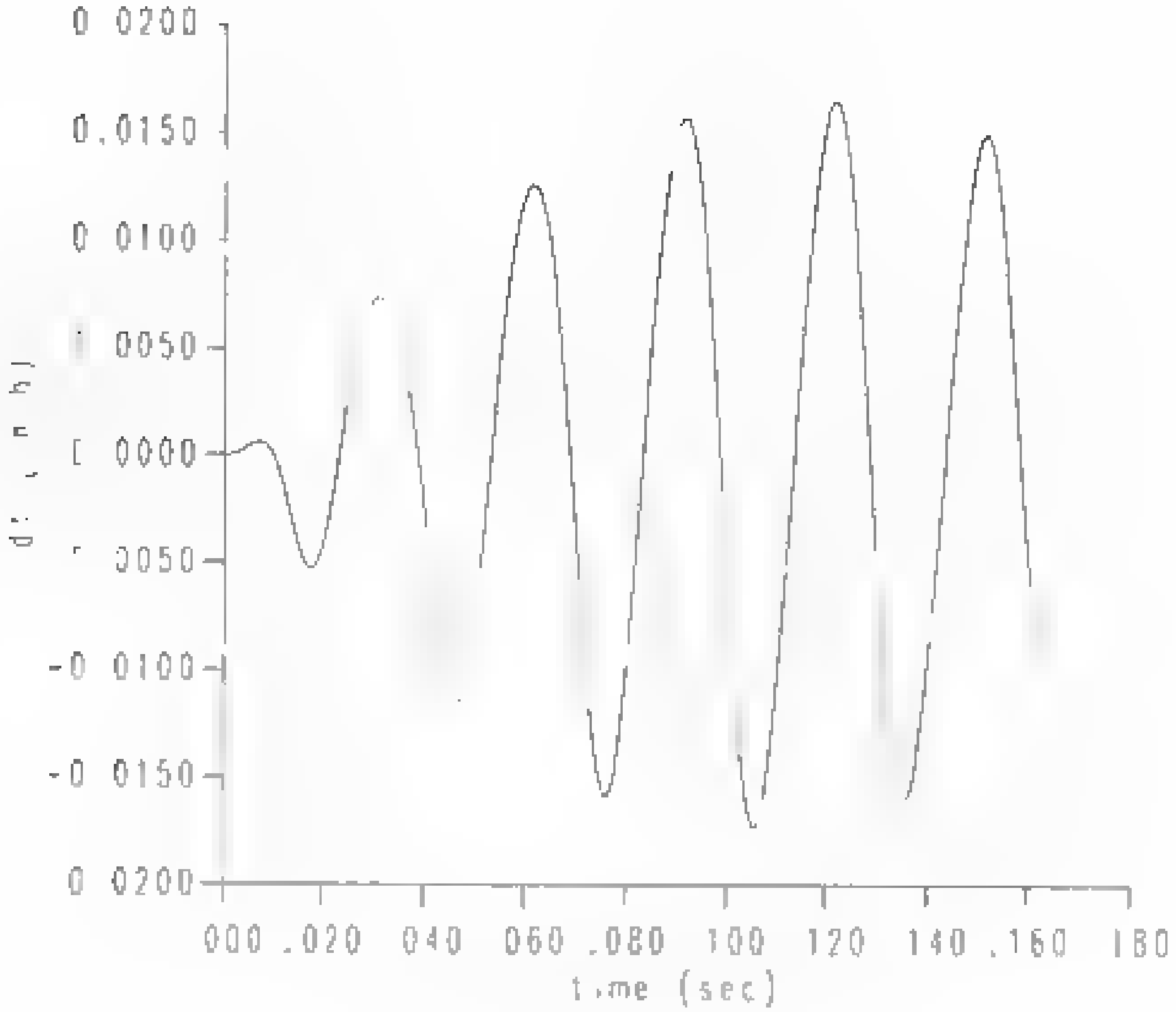


Fig. 5.3 Deflection of Gear Tooth at the Mesh due to an Unbalance Excitation at Rotor 2 with the Effect of Oil Film Bearings (Case 30) $\tau_{ex} = .02$, $\tau_{no} = .02$

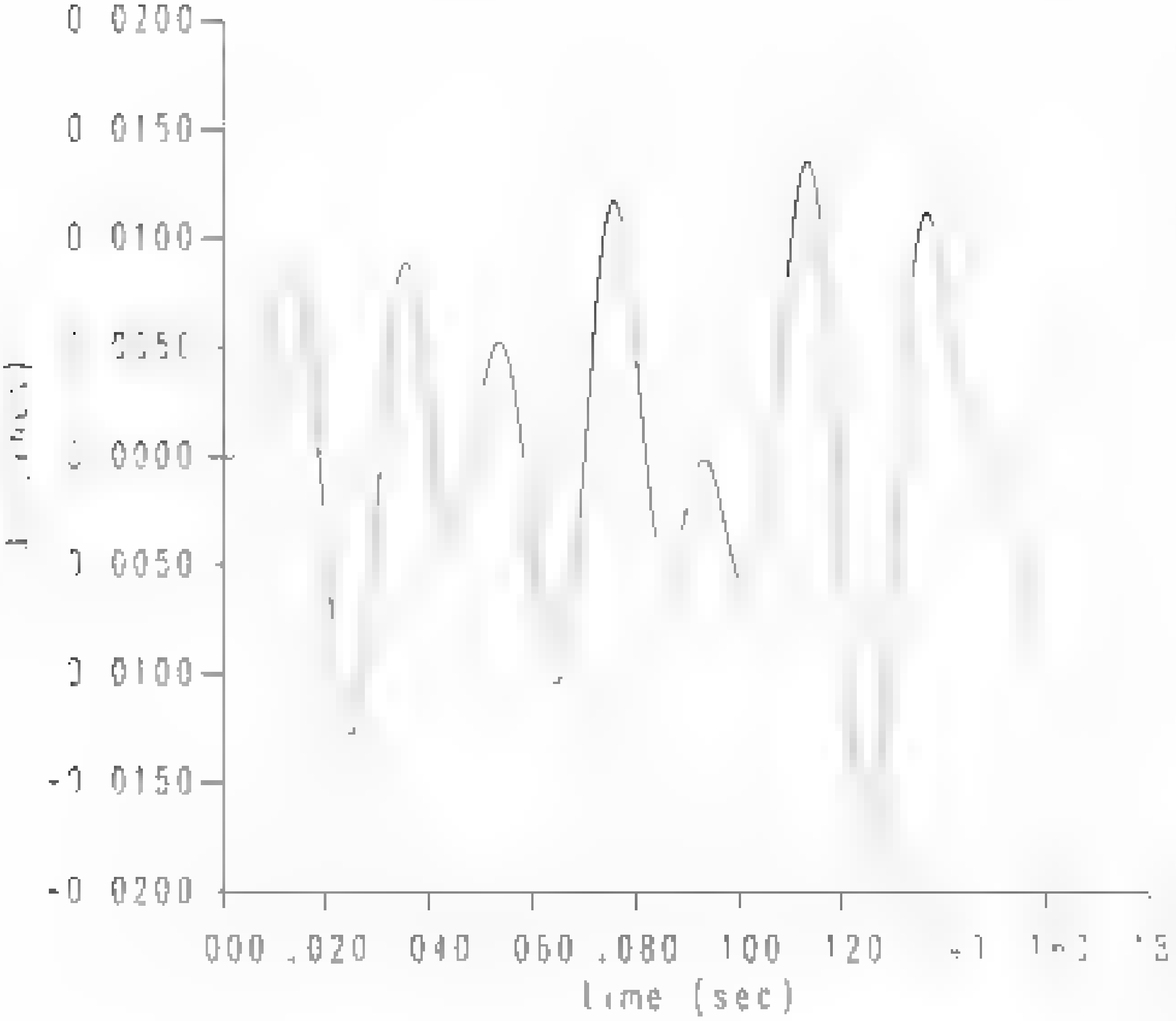


Fig. 5.4 Deflection of Gear Tooth at the Mesh due to the Two Excitations with Proper Phase Shift with the Effect of Oil Film Bearings (Case 30)
$\tau_{ex} = .02$, $\tau_{no} = .02$

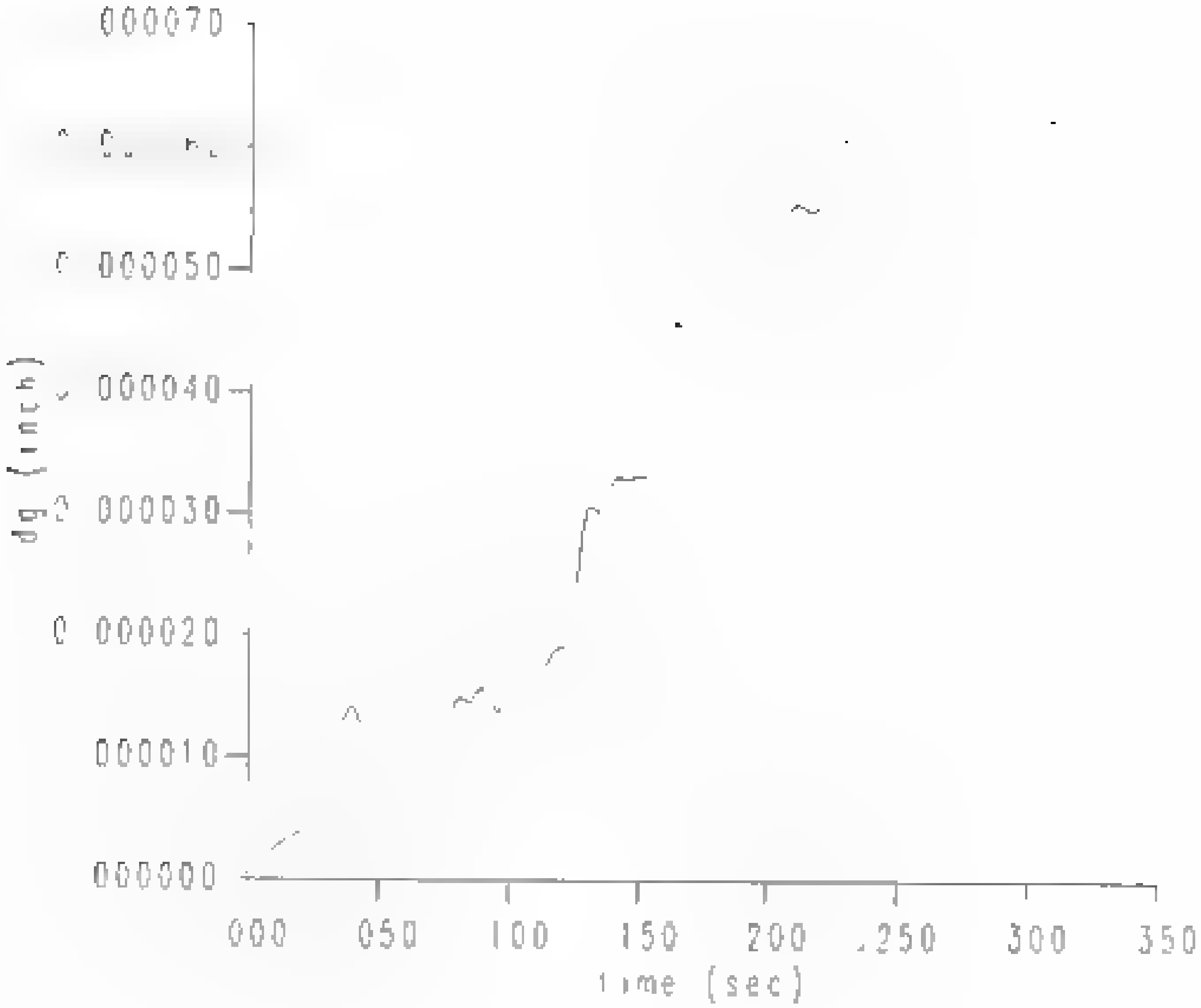


Fig. 5.5 Gear Whirl for Case 30
$\tau_{ex} = .02$, $\tau_{no} = .02$

CHAPTER 6
GUIDELINES FOR SYSTEM DESIGN

Based on the results of the reported study, the following is a suggested procedure for guiding the designer in selecting the system parameters which insure that the response of the system remains within acceptable limits.

Stable System with Torque Excitation

1. Calculate the system response for the given operating condition. If the response exhibits no build-up, and the vibration amplitude is acceptable, no more treatment will be necessary.

2. If the system is stable and the amplitude is more than can be tolerated, the use of fluid film bearing with appropriately selected parameters can be investigated as a means of reducing the vibration to an acceptable level. This is accomplished by selecting the parameters of the fluid film bearings supporting the shafts in such a way as to introduce positive damping in the system. This is done by changing the bearing length, the radial clearance, and the viscosity iteratively until the oscillation is suppressed to the

allowable limit. This is illustrated by the results of the Cases 19, 20. The introduction of a fluid film bearing with l_b = 2.5", c = .005", $\mu = 1.36/10^8$ reyns suppressed 20 % of the amplitude of deflection of the gear tooth (from .0004" to .00032"), and eliminated the beat from the response as shown in Figs. 4.1 and 4.2.

3. If necessary, further reduction of the system response can be achieved by imposing an additional "tuned" excitation on the other rotor of the system supported on the oil film bearing selected in the last step. If it is possible to introduce a torque excitation in the system, an additional torque with same magnitude and 180 degrees phase shift can practically eliminate the system oscillation. This is illustrated in Cases 2, 3. The introduction of the same amount of torque with 180 degrees of phase shift suppressed 99 % of the amplitude of deflection of gear tooth (from .0003" to .000003") as shown in Figs 4.2 and 3.3. In case additional torque excitation is not practical, the introduction of an unbalance on the other gear with a phase shift which is determined by matching the peak amplitude of the responses due to the individual excitations can further reduce the system response. This is illustrated in Cases 19, 21. The introduction of an unbalance excitation corresponding to an eccentricity with e_1 = .00053" with 180 degrees of phase shift suppressed 90 % of the amplitude of deflection of the gear tooth (from .0003" to .00003") as shown in Figs. 4.2, 4.4.

Resonant System with Torque Excitation

1. The reduction of the system response can be achieved by imposing an additional "tuned" excitation on the other rotor. If it is possible to introduce a torsional excitation in the system, an additional torque with same magnitude and 180 degrees phase shift can reduce the system response drastically. This is illustrated in Cases 4, 5. The introduction of the same amount of torque with 180 degrees of phase shift eliminated 99 % of the amplitude of deflection of the gear tooth (from .0015" to .000002") at t =.007 sec. as shown in Figs. 3.5 and 3.6.

2. The introduction of a fluid film bearing can completely eliminate any residual effect. The procedure of the selection of the parameters of the oil film bearing is the same as discussed in the stable case. This is illustrated in Cases 5, 6. As shown in Figs. 3.6 and 3.7, the oil film bearing with l_b = 2.5", c = .006", $\mu = 1.36/10^8$ reyns reduced the amplitude of deflection of the gear tooth to 0.02 % of the original amplitude (from .000002" to .0000003"), and eliminated the build-up.

In case it is not possible to impose an additional torque on the system, the use of an unbalance in the other gear with a phase shift which is determined by matching the peak amplitude of the responses due to individual excitations can reduce the system response. In this case, the effect of oil film bearings on the system response subject to each excitation is different. Accordingly, the individual response of the system supported on the selected bearings due to each excitation is obtained first to decide the appropriate amount of phase shift and unbalance.

The magnitude of unbalance is determined by matching the peak amplitude and the amount of phase shift is selected in such a way as to make the peak responses out of phase with each other. This is illustrated in Cases 22, 23, and 24. The response of the system supported on the appropriately selected bearings due to torsional and unbalance excitations respectively is shown in Figs. 4.7 and 4.8. The introduction of an unbalance excitation corresponding to an eccentricity e_3 = .000012" with 61.9 degrees of phase lag along with properly designed bearings (l_b = 2.5", c = .006", $\mu = .5/10^8$) suppressed 95 % of the amplitude of deflection of gear tooth (from .002" to .0001"), and eliminated the build-up as shown in Figs. 4.6 and 4.9.

Unstable System with Torque Excitation

1. The stabilization of the system response with exponential build-up can also be achieved by imposing an additional "tuned" excitation on the other rotor. If it is possible to introduce a torsional excitation in the system, an additional torque with the same magnitude and 180 degrees

phase shift can reduce the system response drastically. This is illustrated in Cases 7, 8. The introduction of the same amount of torque with 180 degrees of phase shift eliminated 99.9 % of the amplitude of deflection of the gear tooth (from .005" to .000003") at t =.007 sec. as shown in Figs. 3.9 and 3.10.

2. The introduction of a fluid film bearing can further reduce the response. Again the parameters of the oil film bearings are selected in such a way to provide the system with positive damping as discussed in the previous cases. This is illustrated in Cases 8, 9. As shown in Figs. 3.10 and 3.11, the oil film bearing with l_b = 2.5", c = .004", $\mu = 1.36/10^8$ reyns can further suppress the deflection of the gear tooth.

Once again, in case it is not possible to impose an additional torque on the system, the introduction of an unbalance on the other rotor can reduce the system response. The procedure for selection of the magnitude of unbalance and the amount of phase shift is the same as in the resonant case. This is illustrated in Cases 25, 26, and 27. The response of the system supported on the appropriately selected bearings due to torsional and unbalance excitations respectively is shown in Figs. 4.7 and 4.8. The introduction of an unbalance excitation equivalent to an eccentricity e_3 = .00006" with a 32.8 degrees of phase lag along with properly designed bearings (l_b = 2.5", c = .004", $\mu = 1.36/10^8$ reyns) suppressed 95 % of the amplitude of deflection of gear tooth (from .004" to .0002"), and eliminated the build-up as shown in Figs. 4.11 and 4.14.

Stable System with Unbalance Excitation

1. Calculate the system response for the given operating condition. If the response exhibits no build-up, and the vibration amplitude is acceptable, no more treatment will be necessary.

2. If the system is stable and the amplitude is more than can be tolerated, the reduction of the system response can be achieved by imposing an additional "tuned" excitation on the other rotor. An additional unbalance excitation with the same magnitude and 0 degree phase shift can further reduce the system response. This is illustrated in Cases 10, 11. The introduction of the same amount of unbalance with 0 degree of phase shift suppressed approximately 90 % of the amplitude of the high frequency gear whirl (from .0003" to .00003") as shown in Figs 3.13 and 3.14.

3. The introduction of fluid film bearings selected in an iterative way to supply the proper amount of damping to the system can further stabilize the system response. This is illustrated in Cases 11, 12. As shown in Figs. 3.14 and 3.15, the oil film bearing with $l_b = 2.5"$, $c = .005"$, $\mu = 1.36/10^8$ reyns eliminated 95 % of the fluctuation in gear whirl (from .0004" to .000002").

Lateral Resonance due to Unbalance Excitation

1. The reduction of the system response can be achieved by imposing an additional "tuned" unbalance excitation on the other rotor. An additional unbalance excitation with same magnitude and 0 degree phase shift can reduce the system response drastically. This is illustrated in Cases 13, 14. The introduction of the same amount of unbalance with 0 degree of phase shift eliminated approximately 90 % of the amplitude of the high frequency oscillation of gear whirl (from .0012" to .0001") as shown in Figs. 3.17 and 3.18.

2. The introduction of fluid film bearings selected as discussed previously can stabilize the system response. This is illustrated in Cases 14, 15. As shown in Figs. 3.18 and 3.19, the oil film bearings with l_b = 2.5", c = .006", μ = 1.36/10^8 reyns eliminated the build-up, and stabilized the response at .00035".

Subharmonic Resonance due to Unbalance Excitation

1. The reduction of the system response under subharmonic resonance can also be achieved by imposing an additional "tuned" unbalance excitation on the other rotor. An additional unbalance excitation with the same magnitude and 0 degree phase shift can reduce the system response drastically. This is illustrated in Cases 16, 17 The introduction of the same

amount of unbalance with 0 degree of phase shift reduced the amplitude of the high frequency oscillation of gear whirl from .005" to .0006" as shown in Figs. 3.21 and 3.22.

2. The introduction of fluid film bearings which is selected in an iterative way to supply positive damping to the system can enhance the stability and further reduce the response. This is illustrated in Cases 17, 18. As shown in Figs. 3.22 and 3.23, the oil film bearings with $l_b = 2.5"$, $c = .004"$, $\mu = 1.36/10^8$ reyns caused the amplitude of gear whirl to decay.

CHAPTER 7
CONCLUSIONS AND RECOMMENDATIONS

This study demonstrates the feasibility of utilizing lateral and torsional instability properties in multi-rotor geared system to neutralize the potentially destructive response if they occur separately. The use of oil film bearings to stabilize and suppress the amplitude of response has been illustrated by case studies. A procedure is given to guide the designer in eliminating the instability potential in the design stage by utilizing other instabilities and damping in the bearings.

The findings of the reported investigation can be illustrated by the case studies summarized in Table 6.1.

Many cases are considered to illustrate the different possibilities of suppressing the amplitudes of system response. In Chapter 3, additional excitations of the same type as the existing disturbances are imposed on the system to suppress the response.

As shown in section 3.1, the response of the system due to a fluctuating torque excitation on a rotor can be suppressed by introducing additional fluctuating torque input with 180 degrees phase shift in the system at the other rotor.

The response of the system can be further suppressed by the damping provided by appropriate selection of the parameters of oil film bearings. The combined result of imposing an additional input and the appropriate selection of the oil film bearing parameters can cause the system response to exhibit a decay or a limit cycle under conditions of stable, resonant, and unstable system behavior as shown in Cases 3, 6, and 9 respectively.

The possibility of suppressing the system response due to an unbalance on one rotor by an additional unbalance on the other rotor is investigated in section 3.2. An additional unbalance excitation with 0 degree phase shift along with the properly selected oil film bearings can reduce the system response and eliminate the possible instabilities which may occur due to an existing unbalance as shown in Cases 12, 15, and 18 respectively.

The discussions in Chapters 4 and 5 show that it is possible to suppress the effect of an existing disturbance by imposing a "tuned" additional excitation of a different type. It is shown in Chapter 4 that the system response due to an existing torsional excitation on a rotor can be suppressed by an appropriate amount of unbalance imposed on the other rotor along with the appropriate selection of the parameters of the oil film bearings. The amount of unbalance and the phase shift for the unbalance excitation are determined iteratively. An additional tuned excitation of a different type along with the appropriate design of the oil film bearings can stabilize the system and reduce the response amplitude as shown in Cases 21, 24, and 27.

In Chapter 5, it is shown that the same approach can be applied to complex systems with more degrees of freedom. To demonstrate the feasibility of the approach, a worst case condition is investigated (Case 28). The instability due to an existing torsional excitation on an overhang mass is neutralized by imposing an unbalance on the other overhang mass along with appropriate selection of the parameters of oil film bearing as shown in Case 30.

It can be concluded from this study that "tuned" instabilities can be induced in geared systems to provide safe operation at critical speeds where resonances and instabilities may occur. It is therefore possible to design a multi degree of freedom geared system where the build-up of the forces at the mesh can be avoided by appropriate selection of the system parameters. This is a practical approach since both the lateral and the torsional oscillations in gear systems interact at the points of tooth contact.

The suggested procedure in Chapter 6 can be used to guide the designer in selecting the system parameters which insure the stability of the system under the expected critical conditions.

Table 7.1 Effect of the "Tuned" Input and Oil Film Bearings

Excitation	Initial response	Induced change	Final response
T_1=1500 ω_{ex}=3750 (ω_{no}=5012)	Limit cycle d_t=.0004" (Fig.3.1)	$T_3=-T_1$ c=.005 $\mu=1.36/10^6$	limit cycle d_t=4E-07" (Figs.3.3,3.4)
T_1=1500 ω_{ex}=5000 (ω_{no}=5012)	Linear build-up d_t=.25 (t) (Fig. 3.5)	$T_3=-T_1$ c=.006 $\mu=1.36/10^6$	decay (Figs. 3.7, 3.8)
T_1=1500 ω_{ex}=2500 (ω_{no}=5012)	Expon. build-up $d_t=e^{.5t}-1$ (Fig. 3.9)	$T_3=-T_1$ c=.004 $\mu=1.36/10^6$	limit cycle d_t=3E-06" (Figs. 3.11, 12)
Me_1=3E-05 ω_r=3750 (ω_y=5029)	Oscillating d_g=.0004" (Fig.3.13)	$e_3=e_1$ c=.005 $\mu=1.36/10^6$	constant d_g=.0004" (Figs. 3.15, 16)
Me_1=3E-05 ω_r=5000 (ω_y=5029)	Linear build-up slope =.40 (Fig.3.17)	$e_3=e_1$ c=.006 $\mu=1.36/10^6$	constant d_g=.0004" (Figs. 3.19, 20)
Me_1=3E-05 ω_r=10000 (ω_y=5029)	Linear build-up slope=.02 (Fig. 3.21)	$e_3=e_1$ c=.006 $\mu=1.36/10^6$	constant d_g=.0003" (Figs. 3.23, 24)
T_1=1500 ω_{ex}=3750 (ω_{no}=5012)	Limit cycle d_t=.0004" (Fig.4.1)	Me_3=9E-05 c=.005 $\mu=1.36/10^6$ phase 0	limit cycle d_t=4E-04" (Figs. 4.4, 5)
T_1=1500 ω_{ex}=5000 (ω_{no}=5012)	Linear build-up d_t=.25 (t) (Fig.4.6)	Me_3=1.2E-5 c=.006 $\mu=1.36/10^6$ phase $-.34\pi$	limit cycle d_t=1E-04" (Figs. 4.9, 10)
T_1=1500 ω_{ex}=2500 (ω_{no}=5012)	Expon. build-up $d_t=e^{.5t}-1$ (Fig.4.11)	Me_3=6E-05 c=.004 $\mu=1.36/10^6$ phase $-.18\pi$	limit cycle d_t=2E-04 (Figs. 4.14, 15)

Future Work

The following are recommendations for future work before the proposed approach can be used with certainty in practical applications.

Accurate evaluation of the film parameters is central to the reliable modelling of the whirl motion and is consequently an essential element of the developed procedure. The behavior of the fluid film is assumed to be isoviscous in the reported study. Seireg et al [3, 19, 45] show that the stiffness and damping characteristics can be significantly different when the thermohydrodynamic behavior of the bearing is considered. They suggested an empirical numerical procedure for computing the transient thermohydrodynamic restoring and damping forces generated in the film. This can have considerable effect on the stability threshold as well as the amplitude and frequency of the whirl motion.

Experimental verification of the suggested design procedure is necessary to insure the validity of the approach. This is particularly important in view of the dependency of the analysis on the prediction of the behavior of thin oil films (which are rheological in nature), the out-of-roundness and the surface roughness of the journal and sleeve as well as the thermal and chemical characteristics of the lubricating oil. The thin chemical and absorbed layers generated on the bearing surface in the course of its useful life can also have a significant influence on the behavior of the bearing.

APPENDIX

List of Programs

```
C ********************************************
C *                                          *
C *  TWELVE DEGREES OF FREEDOM GEARED SYSTEM *
C *                                          *
C *        --- MAIN PROGRAM ---              *
C *                                          *
C *     FOR THE  CORNELL SUPER COMPUTER      *
C *                                          *
C ********************************************

C     DOUBLE PRECISION
C

      IMPLICIT REAL*8 (A-H)
      IMPLICIT REAL*8 (O-Z)

      REAL*8       X1(60010),X2(60010),X3(60010),X4(60010)
      REAL*8       Y1(60010),Y2(60010),Y3(60010),Y4(60010)
      REAL*8       TH1(60010),TH2(60010),TH3(60010),TH4(60010)
      REAL*8       DP(61000),DG(61000),DTS(61000)

      REAL I1,I2,J1,J2,J3,J4,NU,IZ1,IZ2,KT1,KT2,INCRE
      REAL L1,L2,L3,L4,L5,L6,KO,K,JEFF

      COMMON WE1,WE2,WE3,WE4,D1,D2,L1,L2,L4,L5,GR
      COMMON BL1,BL2,BD1,BD2,C,W1,W2,N

      OPEN(UNIT=30,STATUS='NEW',FILE='MAIN.DAT')
C
C-----GIVEN PARAMETERS  ---
C-----D'S FOR SHAFTS, R'S FOR ROTORS
C

      ZT=.0
      CONST=.0
      CONV=180/3.14159
      CONV1=386.
      PHI=20/CONV
      C=.002
      D1=2.5
      D2=D1
      BD1=D1+2.*C
      BD2=BD1
      BL1=2.5
      BL2=2.5
      L1=3.0
      L2=16.2
      L3=L2
      L4=L1
      L5=L2
      L6=L5
      R1=5
      R2=R1
      GR=1
      R3=R1/GR
      R4=R3
      WI=3.
      E=30000000
      TQ1=300.
      TQ2=.0
      N=4200
      PSI1=0./CONV
      PSI2=0./CONV
      PSI3=0./CONV
      PSI4=170./CONV
      XI2=0./CONV
      XI4=0./CONV

      W1=N*3.14159*2/60
      W2=GR*W1
      WE1=3.14159*R1**2*WI*.282
      WE2=WE1
      WE3=3.14159*R3**2*WI*.282
      WE4=WE3
      XM1=WE1/CONV1
      XM2=WE2/CONV1
      XM3=WE3/CONV1
      XM4=WE4/CONV1
      E1=.0/XM1
      E3=.0/XM3
      E2=.0/XM2
      E4=.08/XM4
      I1=3.14159*D1**4/64
      I2=3.14159*D2**4/64
      J1=XM1*R1**2/2
      J2=XM2*R2**2/2.
      J3=XM3*R3**2/2*GR**2
      J4=XM4*R4**2/2*GR**2.
      NU=.292
      G=E/(2*(1+NU))

C---INFLUENCE COEFFICIENT----

      A11=L2**3/(6*E*I1)
      A12=(L1*L2**2)/(4*E*I1)
      A22=L1**2*(L1+2*L2)/(3*E*I1)
      A33=L5**3/(6*E*I2)
      A34=(L4*L5**2)/(4*E*I2)
      A44=L4**2*(L4+2*L5)/(3*E*I2)

C---CONSTANT RELATED TO THE POSITION OF BEARING-
```

```
      B11=(L1**2+(L1+L2*2)**2)/(L1+L2)**2
      B12=(2*L2**2)/(L1+L2)**2
      B22=B12
      B33=(L4**2+(L4+L5*2)**2)/(L4+L5)**2
      B34=(2*L5**2)/(L4+L5)**2
      B44=B34

C   CONSTANT RELATED TO THE MAG  OF EIGENVALUES

      IX1=1
      IX2=2
      IX3=1
      IX4=2
      IY1=1
      IY2=2
      IY3=1
      IY4=2
      ITH1=1
      ITH2=1
      ITH3=1
      ITH4=1

       IMAX=IX2
       P=950

      P1=P*IX1
      P2=P*IX2
      P3=P*IX3
      P4=P*IX4
      P5=P*IY1
      P6=P*IY2
      P7=P*IY3
      P8=P*IY4
      P9=P*ITH1
      P10=P*ITH2
      P11=P*ITH3
      P12=P*ITH4

C    JJJN DECIDES SAMPLING INTERVAL OF OUTPUT FILE
C    KK DECIDES MULT  OF CALC  TIME BY ADDING DO LOOP(7000)


      JJJ=20000
      JJJN=100
      KK=3

C    ACCR IS ACCURACY CRITERION FOR ITERATION
      ACCR=.0001

      DTMIN=.05*3.14159/180/(P*IMAX)
      INCRE=P*IMAX*DTMIN*180./3.14159
      DTJJ=KK*IMAX*DTMIN

      DT1=DTMIN*IMAX/IX1
      DT2=DTMIN*IMAX/IX2
      DT3=DTMIN*IMAX/IX3
      DT4=DTMIN*IMAX/IX4
      DT5=DTMIN*IMAX/IY1
      DT6=DTMIN*IMAX/IY2
      DT7=DTMIN*IMAX/IY3
      DT8=DTMIN*IMAX/IY4
      DT9=DTMIN*IMAX/ITH1
      DT10=DTMIN*IMAX/ITH2
      DT11=DTMIN*IMAX/ITH3
      DT12=DTMIN*IMAX/ITH4

C    P*DT GIVES THE INCREMENTAL ANGLE ON EACH PHASE PLANE

      KO=4665
      K=KO*R1**2*COS(PHI)**2
      A= █
      JEFF=(J1*J3)/(J1+J3)
      WNO=(K/JEFF)**.5
      WEX=WNO
      WM=20*W1
      IZ1=3.14159*D1**4/32
      IZ2=3.14159*D2**4/32
      KT1=IZ1*G/(L1+L2)
      KT2=IZ2*G/(L4+L5)*GR**2

C    INIT. CONDITIONS
      X1(1)= 0
      X1T=.0
      DX1= 0
      X2(1)=.0
      X2T=.0
      DX2= 0
      X3(1)= 0
      X3T= 0
      DX3= 0
      X4(1)= 0
      X4T= 0
      DX4= 0
      Y1(1)= 0
      Y1T= 0
      DY1= 0
      Y2(1)= 0
      Y2T= 0
      DY2= 0
      Y3(1)= 0
      Y3T= 0
      DY3= 0
```

```
Y4(1)=.0
Y4T= 0
DY4=.0
TH1T=.0
DTH1= 0
TH2(1)=.0
TH2T= 0
DTH2=.0
TH3(1)=.0
TH3T= 0
DTH3= 0
TH4(1)= 0
TH4T= 0
DTH4=.0
XB1= 0
DXB1= 0
XB2=.0
DXB2= 0
XB3=.0
DXB3=.0
XB4=.0
DXB4= 0
YB1=.0
DYB1= 0
YB2=.0
DYB2= 0
YB3=.0
DYB3=.0
YB4=.0
DYB4=.0
THB1=.0
DTHB1= 0
THB2= 0
DTHB2=.0
THB3= 0
DTHB3= 0
THB4= 0
DTHB4= 0

 DG(1)=(X1(1)**2+Y1(1)**2)** 5
 DP(1)=(X3(1)**2+Y3(1)**2)**.5
 DTS(1)=R1*TH1(1)+R3*TH3(1)

T1=.0
T2=.0
T3=.0
T4= 0
T5= 0
T6= 0
T7= 0
T8= 0
```

```
      T9= 0
      T10= 0
      T11=.0
      T12=.0

C-----CALL SUBROUTINE FOR K'S & C'S DUE TO BEARING-----

      CALL BRNG(S1,S2,XXK1,XXK2,XYK1,XYK2,YXK1,YXK2,YYK1,YYK2,
     *                CXX1,CXX2,CXY1,CXY2,CYX1,CYX2,CYY1,CYY2)

      FRC1= 0
      FRC2= 0

C-----DO LOOP FOR INTEGRATION STEPS BY PHASE PLANE METHOD----

C      THIS DO LOOP(7000) IS MADE JUST TO SEE THE DISPLACEMENTS OF THE
C      SYSTEM LONGER W/O VIOLATING THE VIRTUAL MEMORY SIZE
C      X(JJ+1) IS THE DISPLACEMENT AFTER KK TIMES OF CALCULATION
C      (EACH WITH INCREMENTAL ANGLE P*DT) SINCE X(JJ).

       DO 7000 JJ=1,JJJ

       DO 6000 II=1,KK

       DO 6100 MM=1,IX1

       SM1=KG*(1-A*COS(WM*T1))
       DM1=ZT*2*(SM1*XM3*XM1/(XM3+XM1))**.5

       CALL SIGN(DX1,SGN)

         DTX1=(X1T*(COS(PHI)**2*SM1+XXK1*B11+1/A11)+
     *          DX1*(COS(PHI)**2*DM1+CXX1*B11)+
     *          Y1T*(COS(PHI)*SIN(PHI)*SM5+XYK1*B11)+
     *          DY1*(COS(PHI)*SIN(PHI)*DM5+CXY1*B11)+
     *          TH1T*R1*COS(PHI)**2*SM9+
     *          DTH1*R1*COS(PHI)**2*DM9-
     *          X2T*(XXK1*B12+1/A12)-
     *          DX2*CXX1*B12-
     *          Y2T*XYK1*B12-
     *          DY2*CXY1*B12-
     *          X3T*COS(PHI)**2*SM3-
     *          DX3*COS(PHI)**2*DM3-
     *          Y3T*COS(PHI)*SIN(PHI)*SM7-
     *          DY3*COS(PHI)*SIN(PHI)*DM7+
     *          TH3T*R3*COS(PHI)**2*SM11+
     *          DTH3*R3*COS(PHI)**2*DM11
     *         +SGN*XM1*W1**2*ABS(X1T)*CONST
     *         -XM1*E1*W1**2*COS(W1*T1+PSI1)
     *         -A22/A12*XM2*E2*W1**2*COS(W1*T1+PSI2))
     *          /(XM1*P1**2)-X1T
```

```
            RX1=((X1T+DTX1)**2+(DX1/P1)**2)**.5
            XL1=2*RX1*SIN(P1*DT1/2)
            CALL PSIS(X1T,DX1,DTX1,PSX1,P1)
            CALL PHASE(X1T,DX1,DTX1,PSX1,ANG,SGN1,SGN2,P1,DT1)
                 X1TC=X1T
                 DX1C=DX1
             X1T=X1T+SGN1*XL1*SIN(ANG)
             DX1=(DX1/P1+SGN2*XL1*COS(ANG))*P1

C        ITERATION

 6101   XB1=(X1T+X1TC)/2
         DXB1=(DX1+DX1C)/2

       CALL SIGN(DXB1,SGN)

         DTXB1=(XB1*(COS(PHI)**2*SM1+XXK1*B11+1/A11)+
     *         DXB1*(COS(PHI)**2*DM1+CXX1*B11)+
     *         YB1*(COS(PHI)*SIN(PHI)*SM5+XYK1*B11)+
     *         DYB1*(COS(PHI)*SIN(PHI)*DM5+CXY1*B11)+
     *         THB1*R1*COS(PHI)**2*SM9+
     *         DTHB1*R1*COS(PHI)**2*DM9-
     *         XB2*(XXK1*B12+1/A12)-
     *         DXB2*CXX1*B12-
     *         YB2*XYK1*B12-
     *         DYB2*CXY1*B12-
     *         XB3*COS(PHI)**2*SM3-
     *         DXB3*COS(PHI)**2*DM3-
     *         YB3*COS(PHI)*SIN(PHI)*SM7
     *         DYB3*COS(PHI)*SIN(PHI)*DM7+
     *         THB3*R3*COS(PHI)**2*SM11+
     *         DTHB3*R3*COS(PHI)**2*DM11
     *        +SGN*XM1*W1**2*ABS(XB1)*CONST
     *        -XM1*E1*W1**2*COS(W1*T1+PSI1)
     *        -A22/A12*KM2*E2*W1**2*COS(W1*T1+PSI2))
     *         /(XM1*P1**2)-XB1

           RXB1=((X1TC+DTXB1)**2+(DX1C/P1)**2)**.5
           XLB1=2*RXB1*SIN(P1*DT1/2)
           CALL PSIS(X1TC,DX1C,DTXB1,PSXB1,P1)
           CALL PHASE(X1TC,DX1C,DTXB1,PSXB1,ANG,SGN1,SGN2,P1,DT1)
             XB1=X1TC+SGN1*XLB1*SIN(ANG)
             DXB1=(DX1C/P1+SGN2*XLB1*COS(ANG))*P1
      DLX1=(ABS(X1T)-ABS(XB1))**2+(ABS(DX1)
     *     /P1-ABS(DXB1)/P1)**2
      IF(DLX1. GT. ACCR*(X1T**2+(DX1/P1)**2))THEN
           X1T=XB1
           DX1=DXB1
           GO TO 6101
```

```
        ELSE
          GO TO 6102
        ENDIF

6102         T1=T1+DT1
6100          CONTINUE

      DO 6200 MMM=1,IX2

      CALL SIGN(DX2,SGN)

        DTX2= (-X1T*(XXK1*B12+1/A12)-
    *          DX1*CXX1*B12-
    *          Y1T*XYK1*B12-
    *          DY1*CXY1*B12+
    *          X2T*(XXK1*B22+1/A22)+
    *          DX2*CXX1*B22+
    *          Y2T*XYK1*B22+
    *          DY2*CXY1*B22
    *         +SGN*XM2*W1**2*ABS(X2T)*CONST
    *         -XM2*E2*W1**2*COS(W1*T2+PSI2)
    *         -A11/A12*XM1*E1*W1**2*COS(W1*T2+PSI1))
    *          /(XM2*P2**2)-X2T

             RX2=((X2T+DTX2)**2+(DX2/P2)**2)** 5
             XL2=2*RX2*SIN(P2*DT2/2)
             CALL PSIS(X2T,DX2,DTX2,PSX2,P2)
             CALL PHASE(X2T,DX2,DTX2,PSX2,ANG,SGN1,SGN2,P2,DT2)
                  X2TC=X2T
                  DX2C=DX2
              X2T=X2T+SGN1*XL2*SIN(ANG)
              DX2=(DX2/P2+SGN2*XL2*COS(ANG))*P2

C    ITERATION

6201   XB2=(X2T+X2TC)/2
       DXB2=(DX2+DX2C)/2

        CALL SIGN(DXB2,SGN)

        DTXB2= (-XB1*(XXK1*B12+1/A12)-
    *          DXB1*CXX1*B12-
    *          YB1*XYK1*B.2
    *          DYB1*CXY1*B12+
    *          XB2*(XXK1*B22+1/A22)+
    *          DXB2*CXX1*B22+
    *          YB2*XYK1*B22+
    *          DYB2*CXY1*B22
    *         +SGN*XM2*W1**2*ABS(XB2)*CONST
    *         -XM2*E2*W1**2*COS(W1*T2+PSI2)
    *         -A11/A12*XM1*E1*W1**2*COS(W1*T2+PSI1))
    *          /(XM2*P2**2)-XB2

     RXB2=((X2TC+DTXB2)**2+(DX2C/P2)**2)**.5
     XLB2=2*RXB2*SIN(P2*DT2/2)
     CALL PSIS(X2TC,DX2C,DTXB2,PSXB2,P2)
     CALL PHASE(X2TC,DX2C,DTXB2,PSXB2,ANG,SGN1,SGN2,P2,DT2)
       XB2=X2TC+SGN1*XLB2*SIN(ANG)
       DXB2=(DX2C/P2+SGN2*XLB2*COS(ANG))*P2
     DLX2=(ABS(X2T)-ABS(XB2))**2+(ABS(DX2)
    *     /P2-ABS(DXB2)/P2)**2
     IF(DLX2. GT. ACCR*(X2T**2+(DX2/P2)**2))THEN
         X2T=XB2
         DX2=DXB2
         GO TO 6201
       ELSE
         GO TO 6202
       ENDIF

6202         T2=T2+DT2
6200         CONTINUE

      DO 6300 MMMM=1,IX3

     SM3=KO*(1-A*COS(WM*T3))
     DM3=ZT*2*(SM3*XM3*XM1/(XM3+XM1))**.5

      CALL SIGN(DX3,SGN)

        DTX3=(-X1T*COS(PHI)**2*SM1-
    *          DX1*COS(PHI)**2*DM1-
    *          Y1T*COS(PHI)*SIN(PHI)*SM5-
    *          DY1*COS(PHI)*SIN(PHI)*DM5-
    *          TH1T*R1*COS(PHI)**2*SM9-
    *          DTH1*R1*COS(PHI)**2*DM9+
    *          X3T*(COS(PHI)**2*SM3+XXK2*B33+1/A33)+
    *          DX3*(COS(PHI)**2*DM3+CXX2*B33)+
    *          Y3T*(COS(PHI)*SIN(PHI)*SM7+XYK2*B33)+
    *          DY3*(COS(PHI)*SIN(PHI)*DM7+CXY2*B33)-
    *          TH3T*R3*COS(PHI)**2*SM11-
    *          DTH3*R3*COS(PHI)**2*DM11-
    *          X4T*(XXK2*B34+1/A34)-
    *          DX4*CXX2*B34-
    *          Y4T*XYK2*B34-
    *          DY4*CXY2*B34
    *         +SGN*XM3*W2**2*ABS(X3T)*CONST
    *         -XM3*E3*W2**2*COS(W2*T3+PSI3)
    *         +A44/A34*XM4*E4*W2**2*COS(W2*T3+PSI4))
    *          /(XM3*P3**2)-X3T

     RX3=((X3T+DTX3)**2+(DX3/P3)**2)**.5
```

```
      XL3=2*RX3*SIN(P3*DT3/2)
      CALL PSIS(X3T,DX3,DTX3,PSX3,P3)
      CALL PHASE(X3T,DX3,DTX3,PSX3,ANG,SGN1,SGN2,P3,DT3)
           X3TC=X3T
           DX3C=DX3
        X3T=X3T+SGN1*XL3*SIN(ANG)
        DX3=(DX3/P3+SGN2*XL3*COS(ANG))*P3

C       ITERATION

 6301   XB3=(X3T+X3TC)/2
        DXB3=(DX3+DX3C)/2

       CALL SIGN(DXB3,SGN)

         DTXB3=(-XB1*COS(PHI)**2*SM1-
     *          DXB1*COS(PHI)**2*DM1-
     *          YB1*COS(PHI)*SIN(PHI)*SM5-
     *          DYB1*COS(PHI)*SIN(PHI)*DM5-
     *          THB1*R1*COS(PHI)**2*SM9-
     *          DTHB1*R1*COS(PHI)**2*DM9+
     *          XB3*(COS(PHI)**2*SM3+XXK2*B33+1/A33)+
     *          DXB3*(COS(PHI)**2*DM3+CXX2*B33)+
     *          YB3*(COS(PHI)*SIN(PHI)*SM7+XYK2*B33)+
     *          DYB3*(COS(PHI)*SIN(PHI)*DM7+CXY2*B33)-
     *          THB3*R3*COS(PHI)**2*SM11-
     *          DTHB3*R3*COS(PHI)**2*DM11-
     *          XB4*(XXK2*B34+1/A34)-
     *          DXB4*CXX2*B34-
     *          YB4*XYK2*B34-
     *          DYB4*CXY2*B34
     *         +SGN*XM3*W2**2*ABS(XB3)*CONST
     *         -XM3*E3*W2**2*COS(W2*T3+PSI3)
     *         -A44/A34*XM4*E4*W2**2*COS(W2*T3+PSI4))
     *          /(XM3*P3**2)-XB3

      RXB3=((X3TC+DTXB3)**2+(DX3C/P3)**2)** 5
      XLB3=2*RXB3*SIN(P3*DT3/2)
      CALL PSIS(X3TC,DX3C,DTXB3,PSXB3,P3)
      CALL PHASE(X3TC,DX3C,DTXB3,PSXB3,ANG,SGN1,SGN2,P3,DT3)
        XB3=X3TC+SGN1*XLB3*SIN(ANG)
        DXB3=(DX3C/P3+SGN2*XLB3*COS(ANG))*P3
      DLX3=(ABS(X3T)-ABS(XB3))**2+(ABS(DX3)
     *     /P3-ABS(DXB3)/P3)**2
      IF(DLX3. GT. ACCR*(X3T**2+(DX3/P3)**2))THEN
          X3T=XB3
          DX3=DXB3
          GO TO 6301
        ELSE
          GO TO 6302
        ENDIF
 6302   T3=T3+DT3
 6300   CONTINUE

       DO 6400 MMMMM=1,IX4

       CALL SIGN(DX4,SGN)

         DTX4=(-X3T*(XXK2*B34+1/A34)-
      *        DX3*CXX2*B34-
      *        Y3T*XYK2*B34-
      *        DY3*CXY2*B34+
      *        X4T*(XXK2*B44+1/A44)+
      *        DX4*CXX2*B44+
      *        Y4T*XYK2*B44+
      *        DY4*CXY2*B44
      *       +SGN*XM4*W2**2*ABS(X4T)*CONST
      *       -XM4*E4*W2**2*COS(W2*T4+PSI4)
      *       -A33/A34*XM3*E3*W2**2*COS(W2*T4+PSI3))
      *        /(XM4*P4**2)-X4T

          RX4=((X4T+DTX4)**2+(DX4/P4)**2)**.5
          XL4=2*RX4*SIN(P4*DT4/2)
          CALL PSIS(X4T,DX4,DTX4,PSX4,P4)
          CALL PHASE(X4T,DX4,DTX4,PSX4,ANG,SGN1,SGN2,P4,DT4)
               X4TC=X4T
               DX4C=DX4
           X4T=X4T+SGN1*XL4*SIN(ANG)
           DX4=(DX4/P4+SGN2*XL4*COS(ANG))*P4

C      ITERATION

 6401  XB4=(X4T+X4TC)/2
       DXB4=(DX4+DX4C)/2

       CALL SIGN(DXB4,SGN)

         DTXB4=(-XB3*(XXK2*B34+1/A34)-
      *        DXB3*CXX2*B34-
      *        YB3*XYK2*B34-
      *        DYB3*CXY2*B34+
      *        XB4*(XXK2*B44+1/A44)+
      *        DXB4*CXX2*B44+
      *        YB4*XYK2*B44+
      *        DYB4*CXY2*B44
      *       +SGN*XM4*W2**2*ABS(XB4)*CONST
      *       -XM4*E4*W2**2*COS(W2*T4+PSI4)
      *       -A33/A34*XM3*E3*W2**2*COS(W2*T4+PSI3))
      *        /(XM4*P4**2)-XB4

      RXB4=((X4TC+DTXB4)**2+(DX4C/P4)**2)**.5
      XLB4=2*RXB4*SIN(P4*DT4/2)
      CALL PSIS(X4TC,DX4C,DTXB4,PSXB4,P4)
      CALL PHASE(X4TC,DX4C,DTXB4,PSXB4,ANG,SGN1,SGN2,P4,DT4)
        XB4=X4TC+SGN1*XLB4*SIN(ANG)
        DXB4=(DX4C/P4+SGN2*XLB4*COS(ANG))*P4
      DLX4=(ABS(X4T)-ABS(XB4))**2+(ABS(DX4)
     *     /P4-ABS(DXB4)/P4)**2
      IF(DLX4. GT. ACCR*(X4T**2+(DX4/P4)**2))THEN
         X4T=XB4
         DX4=DXB4
         GO TO 6401
       ELSE
         GO TO 6402
       ENDIF

6402      T4=T4+DT4
6400      CONTINUE

      DO 6500 LL=1,IY1

      SM5=XO*(1-A*COS(WM*T5))
      DM5=ZT*2*(SM5*XM3*XM1/(XM3+XM1))**.5

       CALL SIGN(DY1,SGN)

         DTY1=(X1T*(COS(PHI)*SIN(PHI)*SM1+YXK1*B11)+
     *         DX1*(COS(PHI)*SIN(PHI)*DM1+CYX1*B11)+
     *         Y1T*(SIN(PHI)**2*SM5+YYK1*B11+1/A11)+
     *         DY1*(SIN(PHI)**2*DM5+CYY1*B11)+
     *         TH1T*R1*SIN(PHI)**2*SM9+
     *         DTH1*R1*SIN(PHI)**2*DM9-
     *         X2T*YXK1*B12-
     *         DX2*CYX1*B12-
     *         Y2T*(YYK1*B12+1/A12)-
     *         DY2*CYY1*B12-
     *         X3T*COS(PHI)*SIN(PHI)*SM3-
     *         DX3*COS(PHI)*SIN(PHI)*DM3-
     *         Y3T*SIN(PHI)**2*SM7-
     *         DY3*SIN(PHI)**2*DM7+
     *         TH3T*R3*SIN(PHI)**2*SM11+
     *         DTH3*R3*SIN(PHI)**2*DM11
     *        +SGN*XM1*W1**2*ABS(Y1T)*CONST
     *        -XM1*E1*W1**2*SIN(W1*T5+PSI1)
     *        -A22/A12*XM2*E2*W1**2*SIN(W1*T5+PSI2))
     *         /(XM1*P5**2)-Y1T

              RY1=((Y1T+DTY1)**2+(DY1/P5)**2)**.5
              YL1=2*RY1*SIN(P5*DT5/2)
              CALL PSIS(Y1T,DY1,DTY1,PSY1,P5)
              CALL PHASE(Y1T,DY1,DTY1,PSY1,ANG,SGN1,SGN2,P5,DT5)
                   Y1TC=Y1T
                   DY1C=DY1
               Y1T=Y1T+SGN1*YL1*SIN(ANG)
               DY1=(DY1/P5+SGN2*YL1*COS(ANG))*P5

C       ITERATION

 6501   YB1=(Y1T+Y1TC)/2
        DYB1=(DY1+DY1C)/2

       CALL SIGN(DY1,SGN)

         DTYB1=(XB1*(COS(PHI)*SIN(PHI)*SM1+YXK1*B11)+
     *         DXB1*(COS(PHI)*SIN(PHI)*DM1+CYX1*B11)+
     *         YB1*(SIN(PHI)**2*SM5+YYK1*B11+1/A11)+
     *         DYB1*(SIN(PHI)**2*DM5+CYY1*B11)+
     *         THB1*R1*SIN(PHI)**2*SM9+
     *         DTHB1*R1*SIN(PHI)**2*DM9-
     *         XB2*YXK1*B12-
     *         DXB2*CYX1*B12-
     *         YB2*(YYK1*B12+1/A12)-
     *         DYB2*CYY1*B12-
     *         XB3*COS(PHI)*SIN(PHI)*SM3-
     *         DXB3*COS(PHI)*SIN(PHI)*DM3-
     *         YB3*SIN(PHI)**2*SM7-
     *         DYB3*SIN(PHI)**2*DM7+
     *         THB3*R3*SIN(PHI)**2*SM11+
     *         DTHB3*R3*SIN(PHI)**2*DM11
     *        +SGN*XM1*W1**2*ABS(YB1)*CONST
     *         XM1*E1*W1**2*SIN(W1*T5+PSI1)
     *        -A22/A12*XM2*E2*W1**2*SIN(W1*T5+PSI2))
     *         /(XM1*P5**2)-YB1

      RYB1=((Y1TC+DTYB1)**2+(DY1C/P5)**2)**.5
      YLB1=2*RYB1*SIN(P5*DT5/2)
      CALL PSIS(Y1TC,DY1C,DTYB1,PSYB1,P5)
      CALL PHASE(Y1TC,DY1C,DTYB1,PSYB1,ANG,SGN1,SGN2,P5,DT5)
        YB1=Y1TC+SGN1*YLB1*SIN(ANG)
        DYB1=(DY1C/P5+SGN2*YLB1*COS(ANG))*P5
      DLY1=(ABS(Y1T)-ABS(YB1))**2+(ABS(DY1)
     *      /P5-ABS(DYB1)/P5)**2
      IF(DLY1. GT. ACCR*(Y1T**2+(DY1/P5)**2))THEN
          Y1T=YB1
          DY1=DYB1
          GO TO 6501
        [redacted]
          GO TO 6502
        ENDIF

 6502         T5=T5+DT5
 6500         CONTINUE
```

```
       DO 6600 LLL=1,IY2

       CALL SIGN(DY2,SGN)

         DTY2=(-X1T*YKK1*B12-
     *         DX1*CYX1*B12-
     *         Y1T*(YYK1*B12+1/A12)-
     *         DY1*CYY1*B12+
     *         X2T*YKK1*B22+
     *         DX2*CYX1*B22+
     *         Y2T*(YYK1*B22+1/A22)+
     *         DY2*CYY1*B22
     *        +SGN*XM2*W1**2*ABS(Y2T)*CONST
     *        -XM2*E2*W1**2*SIN(W1*T6+PSI2)
     *        -A11/A12*XM1*E1*W1**2*SIN(W1*T6+PSI1))
     *         /(XM2*P6**2) Y2T

              RY2=((Y2T+DTY2)**2+(DY2/P6)**2)**.5
              YL2=2*RY2*SIN(P6*DT6/2)
              CALL PSIS(Y2T,DY2,DTY2,PSY2,P6)
              CALL PHASE(Y2T,DY2,DTY2,PSY2,ANG,SGN1,SGN2,P6,DT6)
                   Y2TC=Y2T
                   DY2C=DY2
               Y2T=Y2T+SGN1*YL2*SIN(ANG)
               DY2=(DY2/P6+SGN2*YL2*COS(ANG))*P6

C     ITERATION

 6601  YB2=(Y2T+Y2TC)/2
       DYB2=(DY2+DY2C)/2

       CALL SIGN(DYB2,SGN)

         DTYB2=(-XB1*YXK1*B12-
     *         DXB1*CYX1*B12-
     *         YB1*(YYK1*B12+1/A12)-
     *         DYB1*CYY1*B12+
     *         XB2*YXK1*B22+
     *         DXB2*CYX1*B22+
     *         YB2*(YYK1*B22+1/A22)+
     *         DYB2*CYY1*B22
     *        +SGN*XM2*W1**2*ABS(YB2)*CONST
     *        -XM2*E2*W1**2*SIN(W1*T6+PSI2)
     *        -A11/A12*XM1*E1*W1**2*SIN(W1*T6+PSI1))
     *         /(XM2*P6**2)-YB2

       RYB2=((Y2TC+DTYB2)**2+(DY2C/P6)**2)**.5
       YLB2=2*RYB2*SIN(P6*DT6/2)
       CALL PSIS(Y2TC,DY2C,DTYB2,PSYB2,P6)
       CALL PHASE(Y2TC,DY2C,DTYB2,PSYB2,ANG,SGN1,SGN2,P6,DT6)
         YB2=Y2TC+SGN1*YLB2*SIN(ANG)
```

```
      DYB2=(DY2C/P6+SGN2*YLB2*COS(ANG))*P6
    DLY2=(ABS(Y2T)-ABS(YB2))**2+(ABS(DY2)
   *      /P6 ABS(DYB2)/P6)**2
    IF(DLY2. GT. ACCR*(Y2T**2+(DY2/P6)**2))THEN
        Y2T=YB2
        DY2=DYB2
        GO TO 6601
      ELSE
        GO TO 6602
      ENDIF

6602         T6=T6+DT6
6600         CONTINUE

     DO 6700 LLLL=1,IY3

     SM7=KO*(1-A*COS(WM*T7))
     DM7=ZT*2*(SM7*XM3*XM1/(XM3+XM1))**.5

     CALL SIGN(DY3,SGN)

       DTY3=(-X1T*COS(PHI)*SIN(PHI)*SM1-
   *           DX1*COS(PHI)*SIN(PHI)*DM1-
   *           Y1T*SIN(PHI)**2*SM5-
   *           DY1*SIN(PHI)**2*DM5-
   *           TH1T*R1*SIN(PHI)**2*SM9-
   *           DTH1*R1*SIN(PHI)**2*DM9+
   *           X3T*(COS(PHI)*SIN(PHI)*SM3+YXK2*B33)+
   *           DX3*(COS(PHI)*SIN(PHI)*DM3+CYK2*B33)+
   *           Y3T*(SIN(PHI)**2*SM7+YYK2*B33+1/A33)+
   *           DY3*(SIN(PHI)**2*DM7+CYY2*B33)-
   *           TH3T*R3*SIN(PHI)**2*SM11-
   *           DTH3*R3*SIN(PHI)**2*DM11-
   *           X4T*YXK2*B34-
   *           DX4*CYX2*B34-
   *           Y4T*(YYK2*B34+1/A34)-
   *           DY4*CYY2*B34
   *          +SGN*XM3*W2**2*ABS(Y3T)*CONST
   *          -XM3*E3*W2**2*SIN(W2*T7+PSI3)
   *          -A44/A34*XM4*E4*W2**2*SIN(W2*T7+PSI4))
   *           /(XM3*P7**2) Y3T

        RY3=((Y3T+DTY3)**2+(DY3/P7)**2)**.5
        YL3=2*RY3*SIN(P7*DT7/2)
        CALL PSIS(Y3T,DY3,DTY3,PSY3,P7)
        CALL PHASE(Y3T,DY3,DTY3,PSY3,ANG,SGN1,SGN2,P7,DT7)
             Y3TC=Y3T
             DY3C=DY3
         Y3T=Y3T+SGN1*YL3*SIN(ANG)
         DY3=(DY3/P7+SGN2*YL3*COS(ANG))*P7
```

```
C       ITERATION

 6701   YB3=(Y3T+Y3TC)/2
        DYB3=(DY3+DY3C)/2

        CALL SIGN(DYB3,SGN)

          DTYB3=(-XB1*COS(PHI)*SIN(PHI)*SM1-
     *          DXB1*COS(PHI)*SIN(PHI)*DM1-
     *          YB1*SIN(PHI)**2*SM5-
     *          DYB1*SIN(PHI)**2*DM5-
     *          THB1*R1*SIN(PHI)**2*SM9-
     *          DTHB1*R1*SIN(PHI)**2*DM9+
     *          XB3*(COS(PHI)*SIN(PHI)*SM3+YXK2*B33)+
     *          DXB3*(COS(PHI)*SIN(PHI)*DM3+CYX2*B33)+
     *          YB3*(SIN(PHI)**2*SM7+YYK2*B33+1/A33)+
     *          DYB3*(SIN(PHI)**2*DM7+CYY2*B33)-
     *          THB3*R3*SIN(PHI)**2*SM11-
     *          DTHB3*R3*SIN(PHI)**2*DM11-
     *          XB4*YXK2*B34-
     *          DXB4*CYX2*B34-
     *          YB4*(YYK2*B34+1/A34)-
     *          DYB4*CYY2*B34
     *         +SGN*XM3*W2**2*ABS(YB3)*CONST
     *         -XM3*E3*W2**2*SIN(W2*T7+PSI3)
     *         -A44/A34*XM4*E4*W2**2*SIN(W2*T7+PSI4))
     *          /(XM3*P7**2)-YB3

       RYB3=((Y3TC+DTYB3)**2+(DY3C/P7)**2)**.5
       YLB3=2*RYB3*SIN(P7*DT7/2)
       CALL PSIS(Y3TC,DY3C,DTYB3,PSYB3,P7)
       CALL PHASE(Y3TC,DY3C,DTYB3,PSYB3,ANG,SGN1,SGN2,P7,DT7)
         YB3=Y3TC+SGN1*YLB3*SIN(ANG)
         DYB3=(DY3C/P7+SGN2*YLB3*COS(ANG))*P7
       DLY3=(ABS(Y3T)-ABS(YB3))**2+(ABS(DY3)
     *      /P7-ABS(DYB3)/P7)**2
       IF(DLY3. GT. ACCR*(Y3T**2+(DY3/P7)**2))THEN
          Y3T=YB3
          DY3=DYB3
          GO TO 6701
        ELSE
          GO TO 6702
        ENDIF

 6702      T7=T7+DT7
 6700      CONTINUE

       DO 6800 LLLLL=1,IY4

        CALL SIGN(DY4,SGN)
        DTY4=(-X3T*YXK2*B34-
     *          DX3*CYX2*B34-
     *          Y3T*(YYK2*B34+1/A34)-
     *          DY3*CYY2*B34+
     *          X4T*YXK2*B44+
     *          DX4*CYX2*B44+
     *          Y4T*(YYK2*B44+1/A44)+
     *          DY4*CYY2*B44
     *         +SGN*XM4*W2**2*ABS(Y4T)*CONST
     *         -XM4*E4*W2**2*SIN(W2*T8+PSI4)
     *         -A33/A34*XM3*E3*W2**2*SIN(W2*T8+PSI3))
     *          /(XM4*P8**2)-Y4T

         RY4=((Y4T+DTY4)**2+(DY4/P8)**2)**.5
         YL4=2*RY4*SIN(P8*DT8/2)
         CALL PSIS(Y4T,DY4,DTY4,PSY4,P8)
         CALL PHASE(Y4T,DY4,DTY4,PSY4,ANG,SGN1,SGN2,P8,DT8)
               Y4TC=Y4T
               DY4C=DY4
          Y4T=Y4T+SGN1*YL4*SIN(ANG)
          DY4=(DY4/P8+SGN2*YL4*COS(ANG))*P8

C     ITERATION

 6801  YB4=(Y4T+Y4TC)/2
       DYB4=(DY4+DY4C)/2

       CALL SIGN(DYB4,SGN)

         DTYB4=(-XB3*YXK2*B34-
     *          DXB3*CYX2*B34-
     *          YB3*(YYK2*B34+1/A34)-
     *          DYB3*CYY2*B34+
     *          XB4*YXK2*B44+
     *          DXB4*CYX2*B44+
     *          YB4*(YYK2*B44+1/A44)+
     *          DYB4*CYY2*B44
     *         +SGN*XM4*W2**2*ABS(YB4)*CONST
     *         -XM4*E4*W2**2*SIN(W2*T8+PSI4)
     *         -A33/A34*XM3*E3*W2**2*SIN(W2*T8+PSI3))
     *          /(XM4*P8**2)-YB4

      RYB4=((Y4TC+DTYB4)**2+(DY4C/P8)**2)**.5
      YLB4=2*RYB4*SIN(P8*DT8/2)
      CALL PSIS(Y4TC,DY4C,DTYB4,PSYB4,P8)
      CALL PHASE(Y4TC,DY4C,DTYB4,PSYB4,ANG,SGN1,SGN2,P8,DT8)
        YB4=Y4TC+SGN1*YLB4*SIN(ANG)
        DYB4=(DY4C/P8+SGN2*YLB4*COS(ANG))*P8
      DLY4=(ABS(Y4T)-ABS(YB4))**2+(ABS(DY4)
     *     /P8-ABS(DYB4)/P8)**2
      IF(DLY4. GT. ACCR*(Y4T**2+(DY4/P8)**2))THEN
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          Y4T=YB4
          DY4=DYB4
          GO TO 6801
        ELSE
          GO TO 6802
        ENDIF

 6802     T8=T8+DT8
 6800     CONTINUE

        DO 6900 NN=1,ITH1

        SM9=XO*(1-A*COS(WM*T9))
        DM9=ZT*2*(SM9*XM3*XM1/(XM3+XM1))**.5

       CALL SIGN(DTH1,SGN)

          DTTH1=(X1T*R1*COS(PHI)**2*SM1+
     *            DX1*R1*COS(PHI)**2*DM1+
     *            Y1T*R1*SIN(PHI)**2*SM5+
     *            DY1*R1*SIN(PHI)**2*DM5+
     *            TH1T*(R1**2*SM9*COS(PHI)**2+KT1)+
     *            DTH1*R1**2*DM9*COS(PHI)**2-
     *            TH2T*KT1-
     *            X3T*R1*COS(PHI)**2*SM3-
     *            DX3*R1*COS(PHI)**2*DM3-
     *            Y3T*R1*SIN(PHI)**2*SM7-
     *            DY3*R1*SIN(PHI)**2*DM7+
     *            TH3T*R1*R3*SM11*COS(PHI)**2+
     *            DTH3*R1*R3*DM11*COS(PHI)**2
     *           +SGN*FRC1*2*WE1*D1/2)
     *            /(J1*P9**2)-TH1T

          RTH1=((TH1T+DTTH1)**2+(DTH1/P9)**2)** 5
          THL1=2*RTH1*SIN(P9*DT9/2)
          CALL PSIS(TH1T,DTH1,DTTH1,PSTH1,P9)
          CALL PHASE(TH.T,DTH1,DTTH1,PSTH1,ANG,SGN1,SGN2,P9,DT9)
               TH1TC=TH1T
               DTH1C=DTH1
           TH1T=TH1T+SGN1*THL1*SIN(ANG)
           DTH1=(DTH1/P9+SGN2*THL1*COS(ANG))*P9

C     ITERATION

 6901  THB1=(TH1T+TH1TC)/2
       DTHB1=(DTH1+DTH1C)/2

       CALL SIGN(DTHB1,SGN)

         DTTHB1=(XB1*R1*COS(PHI)**2*SM1+
     *            DXB1*R1*COS(PHI)**2*DM1+
     *         YB1*R1*SIN(PHI)**2*SM5+
     *         DYB1*R1*SIN(PHI)**2*DM5+
     *         THB1*(R1**2*SM9*COS(PHI)**2+KT1)+
     *         DTHB1*R1**2*DM9*COS(PHI)**2-
     *         THB2*KT1-
     *         XB3*R1*COS(PHI)**2*SM3-
     *         DXB3*R1*COS(PHI)**2*DM3-
     *         YB3*R1*SIN(PHI)**2*SM7-
     *         DYB3*R1*SIN(PHI)**2*DM7+
     *         THB3*R1*R3*SM11*COS(PHI)**2+
     *         DTHB3*R1*R3*DM11*COS(PHI)**2
     *        +SGN*FRC1*2*WE1*D1/2)
     *         /(J1*P9**2)-THB1

      RTHB1=((TH1TC+DTTHB1)**2+(DTH1C/P9)**2)**.5
      THLB1=2*RTHB1*SIN(P9*DT9/2)
      CALL PSIS(TH1TC,DTH1C,DTTHB1,PSTHB1,P9)
      CALL PHASE(TH1TC,DTH1C,DTTHB1,PSTHB1,ANG,SGN1,SGN2,P9,DT9)
        THB1=TH1TC+SGN1*THLB1*SIN(ANG)
        DTHB1=(DTH1C/P9+SGN2*THLB1*COS(ANG))*P9

      DLTH1=(ABS(TH1T)-ABS(THB1))**2+(ABS(DTH1)
     *     /P9-ABS(DTHB1)/P9)**2
      IF(DLTH1  GT  ACCR*(TH1T**2+(DTH1/P9)**2))THEN
         TH1T=THB1
         DTH1=DTHB1
         GO TO 6901
       ELSE
         GO TO 6902
       ENDIF

6902       T9=T9+DT9
6900       CONTINUE

       DO 8000 NNN=1,ITH2

             CALL SIGN(DTH2,SGN)

             DTTH2=(-TH1T*KT1+
    *                  TH2T*KT1-
    *                  TQ1*SIN(WEX*T10+XI2)
    *                  +SGN*FRC1*2*WE1*D1/2)
    *                 /(J2*P10**2)-TH2T

                RTH2=((TH2T+DTTH2)**2+(DTH2/P10)**2)**.5
                THL2=2*RTH2*SIN(P10*DT10/2)
                CALL PSIS(TH2T,DTH2,DTTH2,PSTH2,P10)
      CALL PHASE(TH2T,DTH2,DTTH2,PSTH2,ANG,SGN1,SGN2,P10,DT10)
           TH2TC=TH2T
           DTH2C=DTH2
               TH2T=TH2T+SGN1*THL2*SIN(ANG)
```

```
                 DTH2=(DTH2/P10+SGN2*THL2*COS(ANG))*P10

C       ITERATION

 8001  THB2=(TH2T+TH2TC)/2
       DTHB2=(DTH2+DTH2C)/2

      CALL SIGN(DTHB2,SGN)

         DTTHB2=(-THB1*KT1+
     *              THB2*KT1-
     *              TQ1*SIN(WEX*T10+XI2)
     *              +SGN*FRC1*2*WE1*D1/2)
     *              /(J2*P10**2)-THB2

           RTHB2=(((TH2TC+DTTHB2)/100)**2+(DTH2C/P10/100)**2)**.5*100
           THLB2=2*RTHB2*SIN(P10*DT10/2)
                CALL PSIS(TH2TC,DTH2C,DTTHB2,PSTHB2,P10)
          CALL PHASE(TH2TC,DTH2C,DTTHB2,PSTHB2,ANG,SGN1,SGN2,P10,DT10)
                THB2=TH2TC+SGN1*THLB2*SIN(ANG)
                DTHB2=(DTH2C/P10+SGN2*THLB2*COS(ANG))*P10
             DLTH2=(ABS(TH2T)-ABS(THB2))**2+(ABS(DTH2)
     *              /P10-ABS(DTHB2)/P10)**2
             IF(DLTH2 .GT. ACCR*(TH2T**2+(DTH2/P10)**2))THEN
               TH2T=THB2
               DTH2=DTHB2
               GO TO 8001
             ELSE
               GO TO 8002
             ENDIF

 8002           T10=T10+DT10
 8000           CONTINUE

       DO 8100 NNNN=1,ITH3

      SM11=KO*(1-A*COS(WM*T11))
      DM11=ZT*2*(SM11*XM3*XM1/(XM3+XM1))**.5

      CALL SIGN(DTH3,SGN)

        DTTH3=(X1T*R3*COS(PHI)**2*SM1+
     *           DX1*R3*COS(PHI)**2*DM1+
     *           Y1T*R3*SIN(PHI)**2*SM5+
     *           DY1*R3*SIN(PHI)**2*DM5+
     *           TH1T*R1*R3*SM9*COS(PHI)**2+
     *           DTH1*R1*R3*DM9*COS(PHI)**2-
     *           X3T*R3*COS(PHI)**2*SM3-
     *           DX3*R3*COS(PHI)**2*DM3-
     *           Y3T*R3*SIN(PHI)**2*SM7-
     *           DY3*R3*SIN(PHI)**2*DM7+
     *          TH3T*(R3**2*SM11*COS(PHI)**2+KT2)+
     *          DTH3*R3**2*DM11*COS(PHI)**2-
     *          TH4T*KT2
     *         +SGN*FRC2*2*WE3*D2/2)
     *          /(J3*P11**2)-TH3T

      RTH3=((TH3T+DTTH3)**2+(DTH3/P11)**2)**.5
      THL3=2*RTH3*SIN(P11*DT11/2)
      CALL PSIS(TH3T,DTH3,DTTH3,PSTH3,P11)
      CALL PHASE(TH3T,DTH3,DTTH3,PSTH3,ANG,SGN1,SGN2,P11,DT11)

      TH3TC=TH3T
      DTH3C=DTH3

        TH3T=TH3T+SGN1*THL3*SIN(ANG)
        DTH3=(DTH3/P11+SGN2*THL3*COS(ANG))*P11

C       ITERATION

 8101 THB3=(TH3T+TH3TC)/2
      DTHB3=(DTH3+DTH3C)/2

      CALL SIGN(DTHB3,SGN)

         DTTHB3=(XB1*R3*COS(PHI)**2*SM1+
     *          DXB1*R3*COS(PHI)**2*DM1+
     *          YB1*R3*SIN(PHI)**2*SM5+
     *          DYB1*R3*SIN(PHI)**2*DM5+
     *          THB1*R1*R3*SM9*COS(PHI)**2+
     *          DTHB1*R1*R3*DM9*COS(PHI)**2-
     *          XB3*R3*COS(PHI)**2*SM3-
     *          DXB3*R3*COS(PHI)**2*DM3-
     *          YB3*R3*SIN(PHI)**2*SM7-
     *          DYB3*R3*SIN(PHI)**2*DM7+
     *          THB3*(R3**2*SM11*COS(PHI)**2+KT2)+
     *          DTHB3*R3**2*DM11*COS(PHI)**2-
     *          THB4*KT2
     *         +SGN*FRC2*2*WE3*D2/2)
     *          /(J3*P11**2)-THB3

      RTHB3=((TH3TC+DTTHB3)**2+(DTH3C/P11)**2)**.5
      THLB3=2*RTHB3*SIN(P11*DT11/2)
      CALL PSIS(TH3TC,DTH3C,DTTHB3,PSTHB3,P11)
      CALL PHASE(TH3TC,DTH3C,DTTHB3,PSTHB3,ANG,SGN1,SGN2,P11,DT11)
        THB3=TH3TC+SGN1*THLB3*SIN(ANG)
        DTHB3=(DTH3C/P11+SGN2*THLB3*COS(ANG))*P11
      DLTH3=(ABS(TH3T)-ABS(THB3))**2+(ABS(DTH3)
     *     /P11-ABS(DTHB3)/P11)**2
      IF(DLTH3.GT.ACCR*(TH3T**2+(DTH3/P11)**2))THEN
        TH3T=THB3
        DTH3=DTHB3
        GO TO 8101
       ELSE
        GO TO 8102
       ENDIF

 8102 T11=T11+DT11
 8100 CONTINUE

        DO 8200 NNNNN=1,ITH4

       CALL SIGN(DTH4,SGN)

         DTTH4=( -TH3T*KT2
     *              +TH4T*KT2
     *              +SGN*FRC2*2*WE3*D2/2
     *              -TQ2*SIN(WEX*T12+XI4))
     *              /(J4*P12**2)-TH4T

       RTH4=((TH4T+DTTH4)**2+(DTH4/P12)**2)**.5
       THL4=2*RTH4*SIN(P12*DT12/2)
       CALL PSIS(TH4T,DTH4,DTTH4,PSTH4,P12)
       CALL PHASE(TH4T,DTH4,DTTH4,PSTH4,ANG,SGN1,SGN2,P12,DT12)
            TH4TC=TH4T
            DTH4C=DTH4
         TH4T=TH4T+SGN1*THL4*SIN(ANG)
         DTH4=(DTH4/P12+SGN2*THL4*COS(ANG))*P12

C     ITERATION

 8201    THB4=(TH4T+TH4TC)/2
         DTHB4=(DTH4+DTH4C)/2

       CALL SIGN(DTHB4,SGN)

         DTTHB4=( -THB3*KT2
     *              +THB4*KT2
     *              +SGN*FRC2*2*WE3*D2/2
     *              -TQ2*SIN(WEX*T12+XI4))
     *              /(J4*P12**2)-THB4

       RTHB4=((TH4TC+DTTHB4)**2+(DTH4C/P12)**2)**.5
       THLB4=2*RTHB4*SIN(P12*DT12/2)
       CALL PSIS(TH4TC,DTH4C,DTTHB4,PSTHB4,P12)
       CALL PHASE(TH4TC,DTH4C,DTTHB4,PSTHB4,ANG,SGN1,SGN2,P12,DT12)
         THB4=TH4TC+SGN1*THLB4*SIN(ANG)
         DTHB4=(DTH4C/P12+SGN2*THLB4*COS(ANG))*P12
       DLTH4=(ABS(TH4T)-ABS(THB4))**2+(ABS(DTH4)
     *      /P12-ABS(DTHB4)/P12)**2
       IF(DLTH4 .GT. ACCR*(TH4T**2+(DTH4/P12)**2))THEN
           TH4T=THB4
           DTH4=DTHB4
```

```
          GO TO 8201
        ELSE
          GO TO 8202
        ENDIF

8202    T12=T12+DT12
8200    CONTINUE

6000   CONTINUE

       X1(JJ+1)=X1T
       X2(JJ+1)=X2T
       X3(JJ+1)=X3T
       X4(JJ+1)=X4T
       Y1(JJ+1)=Y1T
       Y2(JJ+1)=Y2T
       Y3(JJ+1)=Y3T
       Y4(JJ+1)=Y4T
       TH1(JJ+1)=TH1T
       TH2(JJ+1)=TH2T
       TH3(JJ+1)=TH3T
       TH4(JJ+1)=TH4T
          DTS(JJ+1)=R1*TH1(JJ+1)+R3*TH3(JJ+1)
          DG(JJ+1)=(X1(JJ+1)**2+Y1(JJ+1)**2)**.5
          DP(JJ+1)=(X3(JJ+1)**2+Y3(JJ+1)**2)**.5

7000             CONTINUE


     WRITE(30,919)N,GR,BD1,L1,WNO,WEX
 919 FORMAT('RPM=',1I6,1X,'GEAR RAT.=',1F4.1,1X,'BD=',1F5.3,
   *        1X,'L1=',1F6.3,1X,'WNO=',1F10.3,1X,'WEX=',1F10.3)
     WRITE(30,*)
     WRITE(30,309) INCRE,ACCR
 309 FORMAT(2X,'INCR. ANGLE=',1F11.8,5X,'AC  CR =',1F9.6)

     WRITE(30,*)
     WRITE(30,319)
 319 FORMAT(4X,'TIME',18X,'X1',15X,'SAMPLING')
     TJJ= 0
     DO 311 JJ=1,JJJ,JJJN
     WRITE(30,312) TJJ,X1(JJ),JJ
 312 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
     TJJ=TJJ+DTJJ*JJJN
 311 CONTINUE

     WRITE(30,*)
     WRITE(30,319)
 319 FORMAT(4X,'TIME',18X,'X1',15X,'SAMPLING')
     TJJ= 0
```

```
      DO 311 JJ=1,JJJ,JJJN
      WRITE(30,312) TJJ,X1(JJ),JJ
  312 FORMAT(1F11.7,6X,1E18 9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  311 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,329)
  329 FORMAT(4X,'TIME',18X,'X2',15X,'SAMPLING')
      TJJ= 0
      DO 322 JJ=1,JJJ,JJJN
      WRITE(30,323) TJJ,X2(JJ),JJ
  323 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  322 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,339)
  339 FORMAT(4X,'TIME',18X,'X3',15X,'SAMPLING')
      TJJ= 0
      DO 333 JJ=1,JJJ,JJJN
      WRITE(30,334) TJJ,X3(JJ),JJ
  334 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  333 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,349)
  349 FORMAT(4X,'TIME',18X,'X4',15X,'SAMPLING')
      TJJ=.0
      DO 344 JJ=1,JJJ,JJJN
      WRITE(30,345) TJJ,X4(JJ),JJ
  345 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  344 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,359)
  359 FORMAT(4X,'TIME',18X,'Y1',15X,'SAMPLING')
      TJJ=.0
      DO 355 JJ=1,JJJ,JJJN
      WRITE(30,356) TJJ,Y1(JJ),JJ
  356 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  355 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,369)
  369 FORMAT(4X,'TIME',18X,'Y2',15X,'SAMPLING')
      TJJ=.0
      DO 366 JJ=1,JJJ,JJJN
      WRITE(30,367) TJJ,Y2(JJ),JJ
  367 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  366 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,379)
  379 FORMAT(4X,'TIME',18X,'Y3',15X,'SAMPLING')
      TJJ=.0
      DO 377 JJ=1,JJJ,JJJN
      WRITE(30,378) TJJ,Y3(JJ),JJ
  378 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  377 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,389)
  389 FORMAT(4X,'TIME',18X,'Y4',15X,'SAMPLING')
      TJJ=.0
      DO 388 JJ=1,JJJ,JJJN
      WRITE(30,387) TJJ,Y4(JJ),JJ
  387 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  388 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,397)
  397 FORMAT(4X,'TIME',18X,'TH1',14X,'SAMPLING')
      TJJ=.0
      DO 399 JJ=1,JJJ,JJJN
      WRITE(30,398) TJJ,TH1(JJ),JJ
  398 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  399 CONTINUE

      WRITE(30,*)
      WRITE(30,*)
      WRITE(30,409)
  409 FORMAT(4X,'TIME',18X,'TH2',14X,'SAMPLING')
      TJJ=.0
      DO 400 JJ=1,JJJ,JJJN
      WRITE(30,401) TJJ,TH2(JJ),JJ
  401 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
```

```
     TJJ=TJJ+DTJJ*JJJN
 400 CONTINUE

     WRITE(30,*)
     WRITE(30,*)
     WRITE(30,419)
 419 FORMAT(4X,'TIME',18X,'TH3',14X,'SAMPLING')
     TJJ= 0
     DO 411 JJ=1,JJJ,JJJN
     WRITE(30,412) TJJ,TH3(JJ),JJ
 412 FORMAT(1F11.7,6X,1E18.9,10X,I16)
     TJJ=TJJ+DTJJ*JJJN
 411 CONTINUE

     WRITE(30,*)
     WRITE(30,*)
     WRITE(30,429)
 429 FORMAT(4X,'TIME',18X,'TH4',14X,'SAMPLING')
     TJJ= 0
     DO 422 JJ=1,JJJ,JJJN
     WRITE(30,423) TJJ,TH4(JJ),JJ
 423 FORMAT(1F11.7,6X,1E18.9,10X,I16)
     TJJ=TJJ+DTJJ*JJJN
 422 CONTINUE

     WRITE(30,*)
     WRITE(30,719)
 719 FORMAT(4X,'TIME',18X,'DP',15X,'SAMPLING')
     TJJ= 0
     DO 711 JJ=1,JJJ,JJJN
     WRITE(30,712) TJJ,DP(JJ),JJ
 712 FORMAT(1F11.7,6X,1E18.9,10X,I16)
     TJJ=TJJ+DTJJ*JJJN
 711 CONTINUE

     WRITE(30,*)
     WRITE(30,819)
 819 FORMAT(4X,'TIME',18X,'DG',15X,'SAMPLING')
     TJJ=.0
     DO 811 JJ=1,JJJ,JJJN
     WRITE(30,812) TJJ,DG(JJ),JJ
 812 FORMAT(1F11.7,6X,1E18.9,10X,I16)
     TJJ=TJJ+DTJJ*JJJN
 811 CONTINUE


     WRITE(30,*)
     WRITE(30,918)
 918 FORMAT(4X,'TIME',18X,'DTS',15X,'SAMPLING')
     TJJ= 0
     DO 911 JJ=1,JJJ,JJJN
```

```
      WRITE(30,912) TJJ,DTS(JJ),JJ
  912 FORMAT(1F11.7,6X,1E18.9,10X,1I6)
      TJJ=TJJ+DTJJ*JJJN
  911 CONTINUE


      STOP
      END
```

```
C ***************************************************
C *             EIGENVALUES                         *
C *                                                 *
C *   CALCULATES EIGENVALUES FOR TWELVE VARIABLES   *
C *                                                 *
C ***************************************************

      IMPLICIT REAL*8 (A-H)
      IMPLICIT REAL*8 (O-Z)

      REAL I1,I2,J1,J2,J3,J4,NU,IZ1,IZ2,KT1,KT2,INCRE,JEFF
      REAL L1,L2,L3,L4,L5,L6,KO,K,KOAVE

      COMMON WE1,WE2,WE3,WE4,D1,D2,L1,L2,L4,L5,GR
      COMMON BL1,BL2,BD1,BD2,C,W1,W2,N

      CHARACTER*8 FN30,FT30
      FN30='EIGEN'
      FT30='DATA'
      CALL CMS
     *('FILEDEF 30 DISK '//FN30//' '//FT30//' * ( LRECL 255 RECFM V',I)

C
C-----GIVEN PARAMETERS-----
C-----D'S FOR SHAFTS, R'S FOR ROTORS
C

      CONV=180/3.14159
      CONV1=386
      PHI=20/CONV
      C=.0063
      D1=2.5
      D2=D1
      BD1=D1+2.*C
      BD2=BD1
      BL1=2.5
      BL2=2.5
      L1=3.0
C     L2=14
       L2=4.325
      L3=L2
      L4=3.0
C     L5=14
       L5=L2
      L6=L5
      R1=7.0
      R2=R1
       GR=1
      R3=R1/GR
      R4=R3
       W1=3./2
      E=30000000
      NU=.292
      G=E/(2*(1+NU))
      TQ=WI*1000
      N=47750
      W1=N*3.14159*2/60
      W2=GR*W1
      WE1=3.14159*R1**2*WI*.282
      WE2=WE1
      WE3=3.14159*R3**2*WI*.282
      WE4=WE3
      XM1=WE1/CONV1
      XM2=WE2/CONV1
      XM3=WE3/CONV1
      XM4=WE4/CONV1

      I1=3.14159*D1**4/64
      I2=3.14159*D2**4/64

C---INFLUENCE COEFFICIENT-------

      A11=L2**3/(6*E*I1)
      A12=(L1*L2**2)/(4*E*I1)
      A22=L1**2*(L1+2*L2)/(3*E*I1)
      A33=L5**3/(6*E*I2)
      A34=(L4*L5**2)/(4*E*I2)
      A44=L4**2*(L4+2*L5)/(3*E*I2)


      B11=(L1**2+(L1+L2*2)**2)/(L1+L2)**2
      B12=(2*L2**2)/(L1+L2)**2
      B22=B12
      B33=(L4**2+(L4+L5*2)**2)/(L4+L5)**2
      B34=(2*L5**2)/(L4+L5)**2
      B44=B34

      IZ1=3.14159*D1**4/32
      IZ2=3.14159*D2**4/32
      KT1=IZ1*G/(L1+L2)
      KT2=IZ2*G/(L4+L5)*GR**2
      KO=WI*1.2*1000000
      K=KO*R1**2*COS(PHI)**2

      J1=XM1*R1**2/2
      J2=XM2*R2**2/2
      J3=XM3*R3**2/2*GR**2
      J4=XM4*R4**2/2*GR**2

      JEFF=(J1*J3)/(J1+J3)
      WNO=(K/JEFF)**.5
      WEX=WNO
      Z=20
      WM=Z*W1


C
C ----CALL SUBROUTINE FOR K'S & C'S DUE TO BEARING-----
C


      CALL BRNG(S1,S2,XXK1,XXK2,XYK1,XYK2,YXK1,YXK2,YYK1,YYK2,
     *                 CXX1,CXX2,CXY1,CXY2,CYX1,CYX2,CYY1,CYY2)


       WN1=((COS(PHI)**2*KO+XXK1*B11+1/A11)/XM1)**.5
       WN2=((XXK1*B22+1/A22)/XM2)**.5
       WN3=((COS(PHI)**2*KO+XXK2*B33+1/A33)/XM3)**.5
       WN4=((XXK2*B44+1/A44)/XM4)**.5
       WN5=((SIN(PHI)**2*KO+YYK1*B11+1/A11)/XM1)**.5
       WN6=((YYK1*B22+1/A22)/XM1)**.5
       WN7=((SIN(PHI)**2*KO+YYK2*B33+1/A33)/XM3)**.5
       WN8=((YYK2*B44+1/A44)/XM4)**.5
       WN9=((KT1+R1**2*KO*COS(PHI)**2)/J1)**.5
       WN10=(KT1/J2)**.5
       WN11=((KT2+R3**2*KO*COS(PHI)**2)/J3)**.5
       WN12=(KT2/J4)**.5

       SWN1=((COS(PHI)**2*KO+XXK1*B11+1/A11)/XM1)**.5
       SWN3=((COS(PHI)**2*KO+XXK2*B33+1/A33)/XM3)**.5
       SWN5=((SIN(PHI)**2*KO+YYK1*B11+1/A11)/XM1)**.5
       SWN7=((SIN(PHI)**2*KO+YYK2*B33+1/A33)/XM3)**.5
       SWN9=(R1**2*KO*COS(PHI)**2/J1)**.5
       SWN11=(R3**2*KO*COS(PHI)**2/J3)**.5

C  UNCOUPLED NATURAL FREQUENCIES

       UNF1=(KT1/(J1*J2/(J1+J2)))**.5
       UNF1E=UNF1/1.414213
       UNF2=(KT2/(J3*J4/(J3+J4)))**.5
       UNF21=UNF2/(2**.5)
       UNF3=(K/JEFF)**.5
       UNF35=(KO*COS(PHI)**2/(XM1*XM3/(XM1+XM3)))**.5
       UNF37=(KO*SIN(PHI)**2/(XM1*XM3/(XM1+XM3)))**.5
       UNF39=((UNF35**2+UNF37**2)/2)**.5
       UNF4=(1/(XM1*A11))**.5
       UNF5=((48*E*I2)/((L5+L6)**3)/XM3)**.5
       UNF6=(1/(XM2*A22))**.5
       UNF69=(1/(XM1*A12))**.5
       UNF7=((6*E*I2)/(2*L4**2*(L4+L5+L6))/XM4)**.5
       UNF8=((3*E*I1)/(L1**3)/XM2)**.5
       UNF9=((3*E*I2)/(L4**3)/XM4)**.5
      UNF10=((UNF4**2*XM1+UNF5**2*XM3)/(XM1+XM3))**.5
      UNF11=(XXK1*B11/XM1)**.5
      UNF111=(XXK1*B22/XM2)**.5
      UNF12=(YYK1*B11/XM1)**.5
      UNF121=(YYK1*B22/XM2)**.5
      UNF13=(XXK2*B11/XM3)**.5
      UNF131=(XXK2*B22/XM4)**.5
      UNF14=(YYK2*B11/XM2)**.5
      UNF141=(YYK2*B22/XM4)**.5
      UNF15=((UNF11**2+UNF12**2)/2)**.5
      UNF151=((UNF111**2+UNF121**2)/2)**.5
      UNF16=((UNF13**2+UNF14**2)/2)**.5
      UNF161=((UNF131**2+UNF141**2)/2)**.5

      SUNF1=(2*3*E*I1/L2**3/XM1)**.5

      WRITE(30,11)BD1,BL1,L1,L2,L3,R1
 11   FORMAT('BD1=',1F7.4,2X,'BL1=',1F7.4,2X,'L1=',1F7.4,2X,
     *          'L2=',1F7.4,2X,'L3=',1F7.4,2X,'R1=',1F7.4)
      WRITE(30,112)N,GR,XM1/XM2,KO,K
112   FORMAT(1X,'N=',1I7,2X,'GR=',1F5.2,2X,'MASS RATIO=',1F4.2
     *      ,1X,'KO=',1E11.6,1X,'K=',1E11.6)
      WRITE(30,113)S1,S2,KT1,KT2
113   FORMAT(1X,'S1=',1F10.4,2X,'S2=',1F10.4,2X,'KT1=',1E14.6,2X,'KT2=',
     *      1E14.6)
      WRITE(30,114)XXK1,XXK2,XYK1,XYK2
114   FORMAT('XXK1=',1E11.4,2X,'XXK2=',1E11.4,2X,'XYK1=',1E11.4,2X,
     *      'XYK2=',1E11.4)
      WRITE(30,15)YXK1,YXK2,YYK1,YYK2
 15   FORMAT('YXK1=',1E11.4,2X,'YXK2=',1E11.4,2X,'YYK1=',1E11.4,2X,
     *      'YYK2=',1E11.4)
      WRITE(30,116)CXX1,CXX2,CXY1,CXY2
116   FORMAT('CXX1=',1E11.4,2X,'CXX2=',1E11.4,2X,'CXY1=',1E11.4,2X,
     *      'CXY2=',1E11.4)
      WRITE(30,117)CYX1,CYX2,CYY1,CYY2
117   FORMAT('CYX1=',1E11.4,2X,'CYX2=',1E11.4,2X,'CYY1=',1E11.4,2X,
     *      'CYY1=',1E11.4)
      WRITE(30,119)B11,B12,B22
119   FORMAT(1X,'B11=',1F10.3,3X,'B12=',1F10.3,3X,'B22=',1F10.3)
      WRITE(30,919)1/A11,1/A12,1/A22
919   FORMAT(1X,'1/A11=',1F15.5,3X,'1/A12=',1F15.5,3X,'1/A22=',1F15.5)
      WRITE(30,120)XM1,XM2,XM3,XM4
120   FORMAT(1X,'M1=',1F8.3,3X,'M2=',1F8.3,3X,'M3=',1F8.3,3X,'M4=',1F8.3
     *      )
      WRITE(30,190)J1,J2,J3,J4
190   FORMAT(1X,'J1=',1F7.3,3X,'J2=',1F7.3,3X,'J3=',1F7.3,3X,'J4=',1F7.3
     *      )
      WRITE(30,*)
      WRITE(30,123)WN1,WN2,WN3,WN4
123   FORMAT(1X,'WN1=',1F11.3,3X,'WN2=',1F11.3,3X,'WN3=',1F11.3,3X,
     *       'WN4=',1F11.3)
```

```
      WRITE(30,124)WN5,WN6,WN7,WN8
  124 FORMAT(1X,'WN5=',1F11.3,3X,'WN6=',1F11.3,3X,'WN7=',1F11.3,3X,
     *        'WN8=',1F11.3)
      WRITE(30,175)WN9,WN10,WN11,WN12
  175 FORMAT(1X,'WN9=',1F11.3,3X,'WN10=',1F11.3,3X,'WN11=',1F11.3,3X,
     *        'WN12=',1F11.3)
      WRITE(30,*)
      WRITE(30,129)WEX,WNO,WM,JEFF
  129 FORMAT(1X,'WEX=',1F10.3,2X,'WNO=',1F10.3,2X,'WM=',1F10.3,2X,
     *        'JEFF=',1F8.5)
      WRITE(30,139)WEX/WNO,WEX/WM,WEX/(Z*W1),W1,W2
  139 FORMAT('WEX/WNO=',1F4.2,1X,'WEX/WM=',1F7.3,1X,'WEX/(Z*W1)='
     *        ,1F7.3,1X,'WE1=',1F8.2,1X,'WE2=',1F8.2)

      WRITE(30,*)
      WRITE(30,501)UNF1
  501 FORMAT(1X,'UNF1 (SHAFT TORSION)            =',1F11.3)
C     WRITE(30,551)UNF1E
C 551 FORMAT(1X,'UNF1E(EQV.TOR.(NEG GEAR INER,1)=',1F11.3)
C     WRITE(30,502)UNF2
C 502 FORMAT(1X,'UNF2 ( 2ND SHAFT TORSION)       =',1F11.3)
C     WRITE(30,592)UNF21
C 592 FORMAT(1X,'UNF21(EQV.TOR.(NEG GEAR INER,2)=',1F11.3)
      WRITE(30,503)UNF3
  503 FORMAT(1X,'UNF3 (TORSIONAL MESH)           =',1F11.3)
      WRITE(30,513)UNF35
  513 FORMAT(1X,'UNF35 (LINEAR MESH, X)          =',1F11.3)
      WRITE(30,523)UNF37
  523 FORMAT(1X,'UNF37 (LINEAR MESH, Y)          =',1F11.3)
      WRITE(30,533)UNF39
  533 FORMAT(1X,'UNF39 (COMBINED LINEAR MESH)    =',1F11.3)
      WRITE(30,504)UNF4
  504 FORMAT(1X,'UNF4 (BENDING  SIMPL SUPP.)     =',1F11.3)
C     WRITE(30,505)UNF5
C 505 FORMAT(1X,'UNF5 (BENDING  SIMPL SUPP.)     =',1F11.3)
      WRITE(30,506)UNF6
  506 FORMAT(1X,'UNF6 (BNDNG  SMPL SUP OVERHNG )=',1F11.3)
      WRITE(30,526)UNF69
  526 FORMAT(1X,'UNF69(BNDG  CTF SMPL SP OVHNG )=',1F11.3)
C     WRITE(30,507)UNF7
C 507 FORMAT(1X,'UNF7 (BNDNG. SMPL SUP OVERHNG )=',1F11.3)
      WRITE(30,508)UNF8
  508 FORMAT(1X,'UNF8 (BNDNG CANTILEVER)         =',1F11.3)
C     WRITE(30,509)UNF9
C 509 FORMAT(1X,'UNF9 (BNDNG CANTILEVER)         =',1F11.3)
C     WRITE(30,510)UNF10
C 510 FORMAT(1X,'UNF10 (BNDNG, COMBINED)         =',1F11.3)
      WRITE(30,511)UNF11
  511 FORMAT(1X,'UNF11 (BEARING SUPPORT, X1,1ST)=',1F11.3)
      WRITE(30,611)UNF111
  611 FORMAT(1X,'UNF111(BEARING SUPPORT, X2,1ST)=',1F11.3)
```

```
      WRITE(30,512)UNF12
  512 FORMAT(1X,'UNF12 (BEARING SUPPORT, Y1,1ST)=',1F11.3)
      WRITE(30,612)UNF121
  612 FORMAT(1X,'UNF121(BEARING SUPPORT, Y2,1ST)=',1F11.3)
C     WRITE(30,513)UNF13
C 513 FORMAT(1X,'UNF13 (BEARING SUPPORT, X1,2ND)=',1F11.3)
C     WRITE(30,613)UNF131
C 613 FORMAT(1X,'UNF131(BEARING SUPPORT, X2,2ND)=',1F11.3)
C     WRITE(30,514)UNF14
C 514 FORMAT(1X,'UNF14 (BEARING SUPPORT, Y1,2ND)=',1F11.3)
C     WRITE(30,614)UNF141
C 614 FORMAT(1X,'UNF141(BEARING SUPPORT, Y2,2ND)=',1F11.3)
      WRITE(30,515)UNF15
  515 FORMAT(1X,'UNF15 (BRNG SUPPORT,1,1CMBINED)=',1F11.3)
      WRITE(30,615)UNF151
  615 FORMAT(1X,'UNF151(BRNG SUPPORT,1,2CMBINED)=',1F11.3)
C     WRITE(30,516)UNF16
C 516 FORMAT(1X,'UNF16 (BRNG SUPPORT,2,1CMBINED)=',1F11.3)
C     WRITE(30,616)UNF161
C 616 FORMAT(1X,'UNF161(BRNG SUPPORT,2,2CMBINED)=',1F11.3)

      WRITE(30,*)
      WRITE(30,901)
  901 FORMAT(1X,'*** SIMPLIFIED SYSTEM ***')

      WRITE(30,*)
      WRITE(30,902)SWN1,SWN3,SWN5
  902 FORMAT(2X,'SWN1=',1F11.3,3X,'SWN3=',1F11.3,3X,'SWN5=',1F11.3,3X)
      WRITE(30,903)SWN7,SWN9,SWN11
  903 FORMAT(2X,'SWN7=',1F11.3,3X,'SWN9=',1F11.3,3X,'SWN11=',1F11.3,3X)

      WRITE(30,*)
      WRITE(30,905)UNF3
  905 FORMAT(1X,'UNF3 (TORSIONAL MESH)           =',1F11.3)
      WRITE(30,904)SUNF1
  904 FORMAT(1X,'SUNF1 (DOUBLE CANTILEVR, X)     =',1F11.3)

      STOP
```

```
C     ********************************************************************
C     *                                                                  *
C     * THIS SUBROUTINE IS TO CALCULATE THE BEARING RELATED PARAMETERS  *
C     * SUCH AS SOMMERFELD NUMBER, STIFFNESS AND DAMPING COEFFICIENTS    *
C     *                                                                  *
C     ********************************************************************

       SUBROUTINE BRNG(S1,S2,XXK1,XXK2,XYK1,XYK2,YXK1,YXK2,YYK1,YYK2,
      *                     CXX1,CXX2,CXY1,CXY2,CYX1,CYX2,CYY1,CYY2)

       IMPLICIT REAL*8 (A-H)
       IMPLICIT REAL*8 (O-Z)

       REAL*8 S(2,500),SN(2),RS(2),RJ(2),BL(2),BD(2)
       REAL*8 PA(2),DTEH(2,500),TMA(2,500),VISCA(2,500)
       REAL*8 PS(2,500),PVISCA(2,500)
       REAL*8 XXKP(2),XYKP(2),YXKP(2),YYKP(2),TOP(2)
       REAL*8 CXXP(2),CXYP(2),CYXP(2),CYYP(2),R(2)

       REAL L1,L2,L3,L4,L5,L6
       COMMON WE1,WE2,WE3,WE4,D1,D2,L1,L2,L4,L5,GR
       COMMON BL1,BL2,BD1,BD2,C,W1,W2,N

C-    --OIL IS SAE20-----
C
C------GIVEN PARAMETERS-----

       WRT1=(WE1+WE2+3.14159/4*D1**2*(L1+2*L2)*.282)/2
       WRT2=(WE3+WE4+3.14159/4*D2**2*(L4+2*L5)*.282)/2
       RS(1)=N/60
       RS(2)=GR*RS(1)
       RJ(1)=D1/2
       RJ(2)=D2/2
       BL(1)=BL1
       BL(2)=BL2
       BD(1)=BD1
       BD(2)=BD2
       TMI=110
       PA(1)=WRT1/(D1*BL1)
       PA(2)=WRT2/(D2*BL2)
       VISCO=1.36/10**8
         PVISCO=VISCO*100000
       R(1)=BL1/BD1
       R(2)=BL2/BD2
         JNC=11203.2
         TOP(1)=.8003+.2783*R(1)
         TOP(2)=.8003+.2783*R(2)


C   -DO LOOP FOR SOMMERFELD NO---
C
```

```
      DO 115 I=1,2

      DTEM(I,1)=.0

      DO 111 J=1,499

      TMA(I,J)=TMI+DTEM(I,J)/2
C     VISCA(I,J)=VISCO*EXP(1271.5/(TMA(I,J)+95))
        PVISCA(I,J)=PVISCO*EXP(1271.5/(TMA(I,J)+95))
C    S(I,J)=VISCA(I,J)*RS(I)/PA(I)*(RJ(I)/C)**2
        PS(I,J)=PVISCA(I,J)*RS(I)/PA(I)*(RJ(I)/C)**2

       IF(R(I). GE. .5)THEN
          IF(PS(I,J). GT. 15000)THEN
           DTEM(I,J+1)=84.2989/(R(I)**.08167)*PS(I,J)**(.8554+
     *              .08787*R(I))*PA(I)/JNC/100000**TOP(I)
         ELSE
           DTEM(I,J+1)=42.0097/(R(I)**.4146)*PS(I,J)**(.6869-
     *              .16*R(I))*PA(I)/JNC/100000*TOP(I)
        ENDIF
       ELSE
          IF(PS(I,J). GT. 15000)THEN
           DTEM(I,J+1)=69.4842/(R(I)**.3075)*PS(I,J)**(.8003+
     *              .2783*R(I))*PA(I)/JNC/100000**TOP(I)
         ELSE
           DTEM(I,J+1)=38.7604/(R(I)**.5307)*PS(I,J)**(.7322-
     *              .2505*R(I))*PA(I)/JNC/100000**TOP(I)
        ENDIF
       ENDIF

      TMA(I,J+1)=TMI+DTEM(I,J+1)/2
C     VISCA(I,J+1)=VISCO*EXP(1271.5/(TMA(I,J+1)+95))
        PVISCA(I,J+1)=PVISCO*EXP(1271.5/(TMA(I,J+1)+95))
C     S(I,J+1)=VISCA(I,J+1)*RS(I)/PA(I)*(RJ(I)/C)**2
        PS(I,J+1)=PVISCA(I,J+1)*RS(I)/PA(I)*(RJ(I)/C)**2

         IF(ABS(PS(I,J+1)-PS(I,J)). LT. .2000 )THEN
            SN(I)=PS(I,J+1)/100000
            GO TO 115
         ELSE
            GO TO 111
         ENDIF

  111 CONTINUE
  115 CONTINUE


C
C  --- DO LOOP FOR K'S & C'S---
C
```

```
      DO 222 K=1,2

      IF(R(K).GE..5)THEN

       IF(SN(K).GE.1.0)THEN
          XXKP(K)=2.3258-1.212*R(K)-.3413*SN(K)+.3436*SN(K)**R(K)
          XYKP(K)=8.1515*R(K)**1.1442*SN(K)**(.4387+.4717*R(K))
          YXKP(K)=12.2356-12.4891*R(K)+1.2669*SN(K)+.0756*SN(K)**2
     *            +2.2224*SN(K)*R(K)-10.9395*SN(K)**R(K)
          YYKP(K)=2.2178*R(K)**(-.1386)*SN(K)**(-.0376+.0383*R(K))
      ELSE
        IF(SN(K).LT..15)THEN
          XXKP(K)=.5979*R(K)**(-1.0181)*SN(K)**(-.8863+.1927*R(K))
          XYKP(K)=2.0501*R(K)**(-.2127)*SN(K)**(-.3713+.1476*R(K))
          YXKP(K)=-.4816+1.7006*R(K)-.9335*SN(K)+11.694*SN(K)**2
     *            -16.3368*SN(K)*R(K)+2.2198*SN(K)**R(K)
          YYKP(K)=2.5181*R(K)**(-.3236)*SN(K)**(.4954-.4007*R(K))
        ELSE
          XXKP(K)=1.1251*R(K)**(-.6746)*SN(K)**(-.8179+.4691*R(K))
          XYKP(K)=7.5105*R(K)**.9778*SN(K)**(-.5584+1.0131*R(K))
          YXKP(K)=1.463+2.044*R(K)-1.29*SN(K)+1.053*SN(K)**2
     *            -12.272*SN(K)*R(K)+4.378*SN(K)**R(K)
          YYKP(K)=2.2202*R(K)**(-.1794)*SN(K)**(.3145-.2771*R(K))
        ENDIF
      ENDIF

      IF(SN(K).GT.1.0)THEN
          CXXP(K)=26.424-25.646*R(K)-11.235*SN(K)-.044*SN(K)**2
     *            +43.391*SN(K)*R(K)-16.947*SN(K)**R(K)
          CXYP(K)=1.353+1.109*R(K)+.376*SN(K)+.027*SN(K)**2
     *            -1.341*SN(K)*R(K)+.510*SN(K)**R(K)
          CYXP(K)=4.059-2.013*R(K)-.231*SN(K) .004*SN(K)**2
     *            +1.839*SN(K)*R(K)-1.450*SN(K)**R(K)
          CYYP(K)=4.735-5.019*R(K) 7.506*SN(K)-.089*SN(K)**2
     *            +26.089*SN(K)*R(K)-2.897*SN(K)**R(K)
      ELSE
        IF(SN(K).GT..25)THEN
          CXXP(K)=6.357-2.298*R(K)-17.294*SN(K)+3.521*SN(K)**2
     *            +19.892*SN(K)*R(K)+5.635*SN(K)**R(K)
          CXYP(K)=.4641+1.8204*R(K)-.6852*SN(K)+.3214*SN(K)**2
     *             2.1412*SN(K)*R(K)+2.2092*SN(K)**R(K)
          CYXP(K)=.4619+2.0240*R(K)-.5145*SN(K)+.4168*SN(K)**2
     *            -2.1685*SN(K)*R(K)+1.9817*SN(K)**R(K)
          CYYP(K)=-7.852+10.414*R(K)-15 .85*SN(K)+3.567*SN(K)**2
     *            +10.084*SN(K)*R(K)+13.869*SN(K)**R(K)
        ELSE
         IF(SN(K).GT..05)THEN
          CXXP(K)=26.402-19.317*R(K) 5.201*SN(K)+55.945*SN(K)**2
     *            +17.102*SN(K)*R(K)-29.361*SN(K)**R(K)
          CXYP(K)=1.7126+.1870*R(K)-9.7244*SN(K)+10.0934*SN(K)**2
     *            +5.0982*SN(K)*R(K)+2.8168*SN(K)**R(K)
```

```
        CYXP(K)=1.6125-.201*R(K)-5.4768*SN(K)+.1344*SN(K)**2
*                +6.7569*SN(K)*R(K)+2.2069*SN(K)**R(K)
        CYYP(K)=1.587-1.083*R(K)-18.793*SN(K)+13.865*SN(K)**2
*                +30.244*SN(K)*R(K)+2.021*SN(K)**R(K)
       ELSE
        CXXP(K)=-8.09+30.59*R(K)-1647.56*SN(K)+7078.46*SN(K)**2
*                +739.86*SN(K)*R(K)+219.81*SN(K)**R(K)
        CXYP(K)=-21.11+27.88*R(K)-705.62*SN(K)+2327.28*SN(K)**2
*                +353.02*SN(K)*R(K)+133.45*SN(K)**R(K)
        CYXP(K)=-22.24+28.43*R(K)-672*SN(K)+2129.06*SN(K)**2
*                +337.77*SN(K)*R(K)+132.83*SN(K)**R(K)
        CYYP(K)=-2.872+5.397*R(K)-204.892*SN(K)+833.223*SN(K)**2
*                +106.01*SN(K)*R(K)+31.329*SN(K)**R(K)

        ENDIF
       ENDIF
      ENDIF

    ELSE

    IF(SN(K). GT. 1.0)THEN
        XXKP(K)=1.2702*R(K)**(-.5053)*SN(K)**(-.8199+.9723*R(K))
        XYKP(K)=5.7477*R(K)**.6401*SN(K)**(.2659+.8173*R(K))
        YXKP(K)=27.92-32.71*R(K)-1.21*SN(K)-.404*SN(K)**2
*                +19.34*SN(K)*R(K)-22.11*SN(K)**R(K)
        YYKP(K)=3.1903*R(K)**.3860*SN(K)**(.1813-.3994*R(K))
   ELSE
     IF(SN(K).LE. .15)THEN

        XXKP(K)=.3532*R(K)**(-1.7179)*SN(K)**(-.2589-1.0922*R(K))
        XYKP(K)=1.9165*R(K)**( .2674)*SN(K)**(-.6257+.6357*R(K))
        YXKP(K) 16.74-26.53*R(K)+13.63*SN(K)-38.70*SN(K)**2
*                +43.72*SN(K)*R(K)-20.12*SN(K)**R(K)
        YYKP(K)=7.4551*R(K)**(1.1975)*SN(K)**(-.6117+1.8354*R(K))
     ELSE
        XXKP(K)=1.1897*R(K)**( .6076)*SN(K)**(-.7152+.3053*R(K))
        XYKP(K)=3 8.02*R(K)**(-.02386)*SN(K)**(-.44757+.85571*R(K))
      IF(SN(K). LE. .5)THEN
        YXKP(K)=4.4333-6.0498*R(K)+3.9980*SN(K)-4.2133*SN(K)**2
*                +.8848*SN(K)*R(K) 4.6921*SN(K)**R(K)
      ELSE
        YXKP(K)=-7.027+10.473*R(K)-1.964*SN(K)+2.039*SN(K)**2
*                -19.4675*SN(K)*R(K)+8.995*SN(K)**R(K)
      ENDIF
        YYKP(K)=3 4.7.*R(K)**.4507*SN(K)**(-.2406+.8095**R(K))
     ENDIF
   ENDIF

   IF(SN(K). GT 1.0)THEN
       CXXP(K) 33.5977-36.630*R(K) 6.205*SN(K) 3.5*SN(K)**2
*               +38.344*SN(K)*R(K)-20.881*SN(K)**R(K)
```

```
          CXYP(K)=4.872-5.657*R(K)-.391*SN(K)+.007*SN(K)**2
     *             +.576*SN(K)*R(K)+.201*SN(K)**R(K)
          CYXP(K)=5.145-4.047*R(K)-.577*SN(K)-.016*SN(K)**2
     *             +2.744*SN(K)*R(K)-1.615*SN(K)**R(K)
          CYYP(K)=3.657-7.447*R(K)-3.246*SN(K)+.174*SN(K)**2
     *             +11.932*SN(K)*R(K)+1.951*SN(K)**R(K)
       ELSE
         IF(SN(K).GT..10)THEN
          CXXP(K)=45.534-57.339*R(K)+5.652*SN(K)-7.791*SN(K)**2
     *             +61.06*SN(K)*R(K)+38.02*SN(K)**R(K)
          CXYP(K)=8.035-10.785*R(K)-.134*SN(K)-.8367*SN(K)**2
     *             +6.204*SN(K)*R(K)-2.591*SN(K)**R(K)
          CYXP(K)=5.348-6.56*R(K)-2.161*SN(K)-.105*SN(K)**2
     *             +5.534*SN(K)*R(K)-.277*SN(K)**R(K)
          CYYP(K)=1.-.6411*R(K)-.7971*SN(K)+.4845*SN(K)**2
     *             +5.3996*SN(K)*R(K)+1.6987*SN(K)**R(K)
         ELSE
          CXXP(K)=151.70-220.05*R(K)+472.18*SN(K)-867.94*SN(K)**2
     *             +59.59*SN(K)*R(K)-224.26*SN(K)**R(K)
          CXYP(K)=9.272-14.117*R(K)-16.803*SN(K)-21.363*SN(K)**2
     *             +44.802*SN(K)*R(K)-1.517*SN(K)**R(K)
          CYXP(K)=.25-.0677*R(K)-64.573*SN(K)+60.894*SN(K)**2
     *             +63.711*SN(K)*R(K)+13.874*SN(K)**R(K)
          CYYP(K)=10.58-14.77*R(K)+63.23*SN(K)-87.31*SN(K)**2
     *             -28.93*SN(K)*R(K)-18.22*SN(K)**R(K)

          ENDIF
         ENDIF
        ENDIF

 222  CONTINUE

      S1=SN(1)
      S2=SN(2)

      XXK1=XXKP(1)*WRT1/C
      XXK2=XXKP(2)*WRT2/C
      XYK1=XYKP(1)*WRT1/C
      XYK2=XYKP(2)*WRT2/C
      YXK1=YXKP(1)*WRT1/C
      YXK2=YXKP(2)*WRT2/C
      YYK1=YYKP(1)*WRT1/C
      YYK2=YYKP(2)*WRT2/C

      CXX1=CXXP(1)*WRT1/(C*W1)
      CXX2=CXXP(2)*WRT2/(C*W2)
      CXY1=CXYP(1)*WRT1/(C*W1)
      CXY2=CXYP(2)*WRT2/(C*W2)
      CYX1=CYXP(1)*WRT1/(C*W1)
      CYX2=CYXP(2)*WRT2/(C*W2)
      CYY1=CYYP(1)*WRT1/(C*W1)
```

```
CYY2=CYYP(2)*WRT2/(C*W2)

RETURN
END
```

```
C ***************************************
C *                                     *
C * SUBROUTINE FOR DECISION OF THE SIGN *
C *                                     *
C ***************************************

      SUBROUTINE SIGN(X,S)

      IMPLICIT REAL*8 (A-H)
      IMPLICIT REAL*8 (O-Z)

      IF(X  GT. .0)THEN
          S=1
      ELSE
        IF(X. EQ. .0)THEN
          S= 0
        ELSE
          S=-1
        ENDIF
      ENDIF

      RETURN
      END
```

```
C  *************************************************************
C  *                                                           *
C  *  SUBROUTINE TO EVALUATE PSI S WHICH WILL BE USED TO DEFINE *
C  *  THE NEXT POSITIONS ON PHASE PLANE                        *
C  *                                                           *
C  *************************************************************

      SUBROUTINE PSIS(DIS,VEL,DELTA,PSI,P)

      IMPLICIT REAL*8 (A-H)
      IMPLICIT REAL*8 (O-Z)

      IF(DIS+DELTA. EQ. .0)THEN
            PSI=3.14159/2
      ELSE
         PSI=ATAN(ABS(VEL)/P/ABS(DIS+DELTA))
      ENDIF

      RETURN
      END
```

```
C  ***********************************************
C  *                                             *
C  *  THIS SUBROUTINE DEFINES THE NEXT POSITIONS *
C  *  ON THE PHASE PLANE                         *
C  *                                             *
C  ***********************************************


      SUBROUTINE PHASE(DIS,VEL,DELTA,PSI,ANG,SGN1,SGN2,P,DT)

      IMPLICIT REAL*8 (A-H)
      IMPLICIT REAL*8 (O-Z)

      IF(DIS+DELTA. GT. .0)THEN
         IF(VEL. GT. .0)THEN
            ANG=PSI-P*DT/2.
            SGN1=1
            SGN2=-1.
         ELSE
            ANG=PSI+P*DT/2
            SGN1=-1
            SGN2=-1
         ENDIF
      ELSE
         IF(DIS+DELTA. LT. .0)THEN
            IF(VEL. GE. .0)THEN
               ANG=PSI+P*DT/2
               SGN1=1
               SGN2=1
            ELSE
               ANG=PSI-P*DT/2
               SGN1= 1
               SGN2=1
            ENDIF
         ELSE
            IF(VEL. GT. .0)THEN
               ANG=3.14159/2 -P*DT/2
               SGN1=1
               SGN2=-1
            ELSE
               IF(VEL. LT. .0)THEN
                  ANG=3.14159/2.-P*DT/2
                  SGN1=-1
                  SGN2=1
               ELSE
                  ANG= 0
                  SGN1= 0
                  SGN2= 0
               ENDIF
            ENDIF
```

```
   ENDIF
ENDIF

RETURN
END
```

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